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A theory of the dynamics of factor shares[☆]

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ABSTRACT

This paper proposes a theory of the dynamics of factor shares within the context of an equilibrium model of endogenous innovation, growth, and cycles. Our deterministic model rests on two assumptions: (i) production requires two complementary inputs, capital, and labor, and (ii) technical progress is labor-saving and embodied in capital goods. The model's unique equilibrium path displays recurring growth cycles, each consisting of an adoption and innovation phase, along which factor shares fluctuate within bounds. The interaction between factor prices and opportunities for labor-saving innovations brings about both persistent growth and aggregate oscillations through which it takes place. We provide evidence that the model-implied correlations between factor shares and the other labor market variables are consistent with the data.

1. Introduction

This paper proposes a theory of the dynamics of factor shares within an equilibrium model of endogenous innovation, growth, and cycles, that is consistent with the main statistical features of aggregate data for the US and other OECD economies.

The mechanism driving the aggregate distribution of national income between capital and labor has been an enduring source of controversy among economists (Hicks, 1932; Kaldor, 1957; Solow, 1958). For long time most macro-economists modeled factor shares as constant both in the long-run and at cyclical frequencies. The work of Piketty (2014) and of many others, have cast doubt on such constancy and revived an interest in equilibrium theories of the dynamics of income distribution.[1] Our paper contributes to this literature in two ways: (i) we show that, in the data, factor shares oscillate quite regularly at medium-run frequencies and, (ii) we propose a model that explains such movements as equilibrium outcomes of the interaction between endogenous innovations and the competitive supply of capital and labor.

Fig. 1.1 shows the gross and net labor share series for the US [1947Q1–2023Q3]. While relative to gross and net domestic product respectively, the long-run trends of the labor share are somewhat different, in both cases, we observe the same wane-and-wax cycles over the medium run (blue curves). By removing business cycle fluctuations using an HP filter, the medium-run fluctuations become

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[1] We briefly review other important contributions at the end of the introduction.

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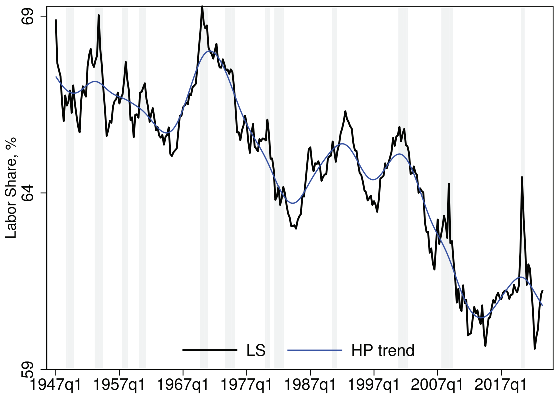
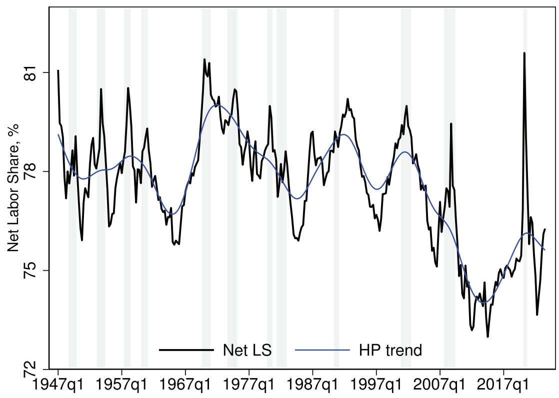


(a) Gross Labor Share



(b) Net Labor Share

Fig. 1.1. Gross and Net Labor Share, 1947Q1–2023Q3.
Note: These figures plot the labor income share in GDP and NDP, respectively, at the quarterly frequency, from 1947Q1 to 2023Q3. Also plotted are the HP trend with a smoothing parameter of 1600 and the NBER dating of recessions.

a robust statistical feature of the labor share series.[2] Which forces drive such persistent oscillations? While a large number of papers have focused on labor share movements either over the long run or at business-cycle frequencies, relatively little is known at the frequencies studied here.

Our theory explains such fluctuations through the interaction between factor prices and opportunities for labor-saving innovations. We address the issue within the context of a model in which a high unit labor cost creates incentives for labor-saving innovations, which reduce the labor income share initially but eventually lead to a full recovery of the productivity gains by the wage earners, which in turn makes the next step innovations profitable. Our model builds on the competitive innovation framework proposed by Boldrin and Levine (2001, 2008), in which production of the final consumption good requires two complementary inputs, capital, and labor. Technical progress is labor-saving and embodied in capital goods: machines of a more recent vintage require less labor to produce one unit of consumption.[3] Each vintage of capital can either reproduce itself or innovate by creating the capital of the next vintage.

The reward from climbing the vintage ladder is a higher return on capital, rather than monopoly power. Because innovation is costly, it becomes profitable to invent the new capital good only when the available quantity of the old capital is large enough to alter relative prices and make it profitable to turn some of the old machines into new and more efficient ones. The evolution of the DRAM semiconductors documented in Irwin and Klenow (1994) vividly portrays the technology process we modeled. Each generation displays a hump-shaped life cycle in shipments. A new quality is introduced when the stock of the old one is fairly large. The old vintage is phased out gradually as the new one is introduced. Further, their price data shows that the price of each vintage of chip falls roughly exponentially over the product cycle — meaning that the incentive to introduce the next generation chip keeps increasing.[4]

While factor prices create incentives for innovations, the labor-saving nature of technical progress affects how capital and labor are rewarded, and does so in a non-monotonic way. When a new technology is adopted, the economy enters the so-called *adoption phase*, during which the new and the previous vintages are simultaneously employed in production. Labor reallocates from the less to the more capital-intensive technology, which increases productivity and reduces the aggregate labor share. Eventually, the old vintage phases out and the new technology absorbs all labor. At that point, it is, however, not profitable to innovate and employ immediately the next even more advanced technology. The rate of return on existing machines is still high, and labor is not expensive

[2] Appendix A provides details on data sources and methods. We have performed the same exercise for other labor share measures studied in the literature: the labor share with and without IPP (Koh et al., 2021) (Figure A.2 in the Appendix), and the labor share with and without the housing and public sector (Gomme and Rupert, 2007; Rognlie, 2015) (Figure A.3). These, as well as the independent work of Bridgman (2018), confirm that the medium-run behavior of the labor share is robust to such adjustments.

[3] Because each new labor-saving *technology* is embodied in *plants/machines* of a new *vintage*, all these words are used interchangeably across the paper.

[4] Figure A.4 in Appendix A presents the prices and shipments for different vintages of dynamic random access memory (DRAM) semiconductors, taken from Irwin and Klenow (1994). This is also closely related to the S-shaped diffusion curve for new products that is widely accepted as a stylized fact. Evidence that factor prices affect (and are affected by) technical change exists in other industries and at the aggregate level. For example, Beckert (2014) documents that in the 18th century wages in the UK are much higher than in other parts of the world like India, which motivates British manufacturers to effect the most momentous technological change in the history of the cotton industry, introducing new machinery, like the spinning jenny, that substantially reduces the unit labor cost. In the same spirit, recent studies highlight both wage increase as a cause, and declining wage share as a result, of automation (Acemoglu and Restrepo, 2018, 2022).

enough, to justify the adoption. The economy then enters the so-called *innovation phase* during which capital of the current vintage accumulates driving its price downward and wages upward, generating an increase in the labor share. Eventually, the changes in relative prices make it profitable to innovate by turning the extra capital stock into new machines embodying a more labor-saving technology, which starts a new cycle.

We analytically solve the model, with either exogenous or endogenous labor supply, and show that during its unique equilibrium path, the model economy settles into such recurring growth cycles, each consisting of an adoption phase and an innovation phase. With no aggregate shocks assumed, the recurring cycles are completely endogenous in our model. Within each cycle, factor shares oscillate but such oscillations average out across cycles. Along with the cycles in factor shares, the model also generates persistent growth in productivity and output and a dynamic correlation pattern among factor shares and other key macroeconomic variables: labor productivity, wages, employment, and output. We go on to show that the model-implied correlations are consistent with the data for both the US and other OECD countries. We close the paper by briefly discussing our assumptions on functional forms, parameter values, and the model's quantitative performance.

Related literature. Recent empirical work (Piketty, 2014; Karabarbounis and Neiman, 2014) documents a decline of labor share in developed countries since the 1980s and particularly after 2000. While the long-run declining-trend hypothesis has been widely questioned under different measurements (Rognlie, 2015; Bridgman, 2018; Koh et al., 2021), that factor shares persistently oscillate over the medium-run is apparent in the data. A few theoretical studies examine (automation) technology and factor shares in equilibrium models with directed technical change (Acemoglu and Restrepo, 2018; Growiec et al., 2018), or with vintage capital and embodied technical progress (Martinez, 2021; Jones and Liu, 2022). In these models the labor share declines after a technology shock, while a self-correcting force – creating new labor-intensive tasks (Acemoglu and Restrepo, 2018), scraping of old technologies (Martinez, 2021), or advances in the productivity of capital inputs (Jones and Liu, 2022) – restores its long-run stability. New technology in our model also leads to initial decline and eventual recovery in the labor share, though via a different mechanism. In our model, the emergence of a new technology is endogenous, driven by profit-seeking choices. In addition, the equilibrium path of existing models displays balanced growth in which factor shares are constant. We contribute to the literature by constructing a model with perpetual growth cycles and factor share oscillations.

The paper is also related to the literature on deterministic growth cycles. Existing models typically consist of a phase with, and a phase without, clustering innovation through strategic complementarity among innovators and short-lived profits due to imitation (Shleifer, 1986), temporary monopoly power (Matsuyama, 1999), or endogenous obsolescence caused by creative destruction (Francois and Lloyd-Ellis, 2003).[5] Our model generates growth cycles through the interaction between factor prices and technical progress. The literature on economic hysteresis also links cycles to growth by capturing recessions' negative and persistent impact on the growth engines in endogenous growth models (Comin and Gertler, 2006; Anzoategui et al., 2019; Ates and Saffie, 2021). A detailed survey can be found in Cerra et al. (2023). In this class of models, exogenous aggregate shocks are needed to generate medium-run cycles, while the cycles are endogenous outcomes in our deterministic framework.[6]

Last, the paper relates to the literature on the countercyclical behavior of factor shares (Boldrin and Horvath, 1995; Gomme and Greenwood, 1995), and on the non-monotonic response of factor shares to exogenous business cycle shocks (Leon-Ledesma and Satchi, 2019; Choi and Rios-Rull, 2020). We capture similar medium run dynamics but endogenize the process.

The rest of the paper is organized as follows: Section 2 outlines the basic model with exogenous labor supply, characterizes the competitive equilibrium, and then studies the implications of endogenous labor supply. Section 3 shows that the model-implied correlations between labor share and other labor market variables are supported by data. The quantitative performance of the model and the role of key parameters and functional forms are discussed in Section 4. Section 5 concludes. Most proofs and calculations are in the appendices.

2. The model

Our model rests on two assumptions: (i) capital and labor are complementary inputs; (ii) technical progress is labor-saving and embodied in capital goods.[7] The environment is standard: recursively complete markets over an infinite horizon and a representative agent with perfect foresight. We consider first the case of exogenous labor supply, with the endogenous case analyzed in Section 2.3.

Preferences The representative household maximizes utility

$$\max \int_0^\infty e^{-\rho t} \log c(t) dt. \tag{1}$$

where $c(t) = \sum_{j=0}^{\infty} c_j(t)$, with $c_j(t)$ the consumption flow from technology j at instant t. The household inelastically supplies one unit of labor.

5 Boldrin and Woodford (1990) provides a survey of the earlier literature. The theoretical model that is closer to our intuition is the prey–predator model of Goodwin (1967), though there is no growth in that model, and economic agents are not optimizing their choices.

6 Along the transitional dynamics, a composition effect should generate oscillations in the aggregate factor share if factor share heterogeneity, e.g. between consumption and capital goods sectors in Comin and Gertler (2006) or high and low type firms in Ates and Saffie (2021), were explicitly modeled.

7 That capital and labor are complementary inputs in the aggregate is supported by most empirical estimates. See e.g. Klump et al. (2007), Oberfield and Raval (2021).

Production Production takes place in three sectors denoted by $s = 1, 2, 3$. Each sector is composed of a continuum of identical firms endowed with capital of some vintage.[8] The first sector produces the consumption goods, the second investment goods, and the third a new vintage of capital embodying a better technology.

Technological Vintages There exists a countably infinite number of potential technologies, indexed by the subscript $j = 0, 1, \ldots$. Technologies are embodied in capital goods: $k_j^s(t)$ denotes the stock of capital embodying technology j installed in sector s at time t. A technology j is active in sector s at time t if $k_j^s(t) > 0$.

Technological Progress A technology with index j is better than a technology with index $j' < j$ for two reasons. First, to produce one unit of consumption, a unit of capital j requires less labor than a unit of capital j', i.e. technological progress is labor-saving. Second, technological progress is incremental insofar as capital $j + 1$ can be obtained, at a cost, only from capital j and not from any other $j' < j$.

Consumption Sector The first sector produces consumption, $c_j(t)$, using capital $k_j^1(t)$ and labor $\ell(t)$ according to a fixed coefficient production function,

$$c_j(t) = \min\{k_j^1(t), \gamma^j \ell(t)\}, \quad \gamma > 1. \tag{2}$$

The assumption that γ is greater than one captures the fact that technological progress is labor-saving: machines of a more advanced vintage require less labor to produce one unit of the consumption good.

Investment Sector The second sector produces additional units of capital of type j from capital of the same vintage:

$$\dot{k}_j(t) = bk_j^2(t), \quad b > \rho. \tag{3}$$

The investment sector allows every kind of capital to self-accumulate at the rate b *after* it has been introduced. The assumption $b > \rho$ means that the rate of capital self-reproduction is larger than the discount rate, which makes accumulation profitable.

Innovation Sector The third sector innovates by producing a new vintage of capital, $j + 1$, from capital of vintage j

$$k_{j+1}(t) = \frac{k_j^3(t)}{a}, \quad a > 1. \tag{4}$$

Assume $a > 1$, hence innovating is costly. The capital stock of type j used in the innovation sector is transformed instantaneously into the new kind of capital $j+1$. Further, capital $j+1$ can be obtained from capital $j', j' < j$ by applying the innovation technology $j - j' + 1$ times, with an innovation ratio of $a^{j'-j-1}$.

Because the stock capital j, k_j, can be employed in any of the three sectors, the following instantaneous resource constraint holds

$$k_j(t) = k_j^1(t) + k_j^2(t) + k_j^3(t). \tag{5}$$

The accumulation equation for capital j, therefore, is

$$dk_j(t) = bk_j^2(t)dt - k_j^3(t) + \frac{k_{j-1}^3(t)}{a}. \tag{6}$$

Note we assume no capital depreciation in the consumption and investment sectors. The investment sector accumulates capital stock continuously, while innovation activities lead to discrete changes.

This economy is an ordinary constant-return economy with three sectors: consumption, investment, and innovation. Diminishing returns to capital accumulation sets in once full employment is reached as capital and labor are complementary inputs in the production of aggregate consumption. As there is perfect competition, the welfare theorems hold and the efficient allocation can be decentralized as a competitive equilibrium, and vice versa. Below we prove that the competitive equilibrium of the economy settles into a sequence of growth cycles, each containing an *adoption* and an *innovation* phase.

2.1. Characterization of the competitive equilibrium

The key step in solving the competitive equilibrium is to establish that at each point in time, there are at most two consecutive vintages of capital employed in production. Use marginal utility as the numeraire, hence the price of consumption at t is $1/c(t)$. Denote with $q_j(t)$ the price of capital j at time t. The physical rate of return in the investment sector is b. Zero profit implies that b plus capital gains must equal the subjective discount rate, $b + \dot{q}_j(t)/q_j(t) = \rho$, or equivalently

$$\dot{q}_j(t)/q_j(t) = -(b - \rho) < 0. \tag{7}$$

This equation implies that the price of capital j decreases as it accumulates over time. The price level and its implications are characterized in the following proposition.

[8] Because firms are identical in each sector, we will talk, indifferently, either of a representative firm with a stock of capital equal to $k(t)$ or of a continuum of identical firms, each one with $k(t)$ units of capital.

Proposition 1. *No more than two vintages of capital are simultaneously used to produce consumption, and they must be consecutive vintages. If j' is used to produce consumption, the price of capital $j, j > j'$ satisfies*

$$q_j(t) \geq \frac{\gamma^{j-j'} - 1}{\gamma^{j-j'} - 1/a^{j-j'}} \frac{1}{bc(t)} \tag{8}$$

where the equality holds if j is also used to produce consumption.

Proof. See Appendix B.1.

Critical in the proof is the computation of the level of capital price. Without loss of generality, assume both capital j' and j, $j > j'$, are used to produce the consumption good. We drop the time argument t for convenience and denote with w, r_j, and $r_{j'}$ the wage rate, and the rental rate of capital j and j', all in units of the final consumption good.[9] The zero profit conditions in the consumption and innovation sectors imply[10]

$$1 - r_j - \frac{w}{\gamma^j} = 0, \quad 1 - r_{j'} - \frac{w}{\gamma^{j'}} = 0, \quad r_j = a^{j-j'} r_{j'}. \tag{9}$$

Solving this system of three equations with three unknowns yields

$$w = \gamma^{j'} \frac{a^{j-j'} - 1}{a^{j-j'} - 1/\gamma^{j-j'}}; \quad r_j = \frac{\gamma^{j-j'} - 1}{\gamma^{j-j'} - 1/a^{j-j'}}; \quad r_{j'} = r_j / a^{j-j'} \tag{10}$$

When machines of types j and j' are simultaneously used in the production of consumption, Eq. (10) implies that the rate of return on capital j is $a^{j-j'} > 1$ times larger than the rate of return on capital j'. Eq. (8) may then be derived by dividing the rental rate of each kind of machine by the rate of accumulation, b, and normalizing by the numeraire, $1/c(t)$.

When capital j' is activated and j, with $j > j'$, is not, we distinguish between the *implicit prices* and *values* of capital j. We use q_j to also denote the implicit price of capital j that would satisfy the zero profit condition of the innovation sector if machines of type j' were converted into machines of type j at the rate $1/a^{j-j'}$. In symbols: $q_j = q_{j'} a^{j-j'}$. Instead, we use the notation $v_j(t)$ to denote the value of capital j if it were employed in the consumption sector, which is equal to the price formula given in Eq. (8). It is important to notice that it is profitable to employ capital j in producing the consumption good at t if only if $q_j(t) \leq v_j(t)$.

This leads to the following characterization of the behavior of consumption.

Proposition 2. *Consumption grows at the rate $b - \rho$ during an adoption phase, which employs two consecutive vintages of capital and lasts for $\tau^d = \frac{\log \gamma}{b - \rho}$ units of time. It is followed by an innovation phase, lasting $\tau^n = \frac{\log a}{b - \rho}$ units of time, during which a single vintage of capital is used in the consumption sector and consumption remains constant. The total length of a cycle is*

$$\tau^* = \frac{\log a + \log \gamma}{b - \rho} \tag{11}$$

Proof. See Appendix B.2.

Consider an adoption phase when capital j and $j+1$ are both used in production, which we refer to as the "$(j, j+1)$ adoption phase". With a fixed labor supply, consumption grows through continual reallocation of labor from capital j to the more productive $j+1$. During this phase, consumption grows at the rate $b - \rho$ by a factor of γ: from γ^j, at the beginning of the phase when all labor is employed in capital j, to γ^{j+1}, at the end when all labor reallocates to capital $j+1$. Hence the adoption phase lasts for $\tau^d = \frac{\log \gamma}{b - \rho}$. At its end, capital $j+1$ employs all the labor force. Diminishing return sets in and there is no incentive to adopt more machines of type $j+1$ in the consumption sector. Shall the economy immediately innovate and adopt technology $j+2$ to enter the next $(j+1, j+2)$ adoption phase?

The answer is NO. At the end of the $(j, j+1)$ adoption phase, the zero profit condition of converting capital $j+1$ to $j+2$ implies that the implicit price of capital $j+2$ equals a times the price of capital $j+1$. We show in Appendix B.2 that, at $t = \tau^d$, the following holds

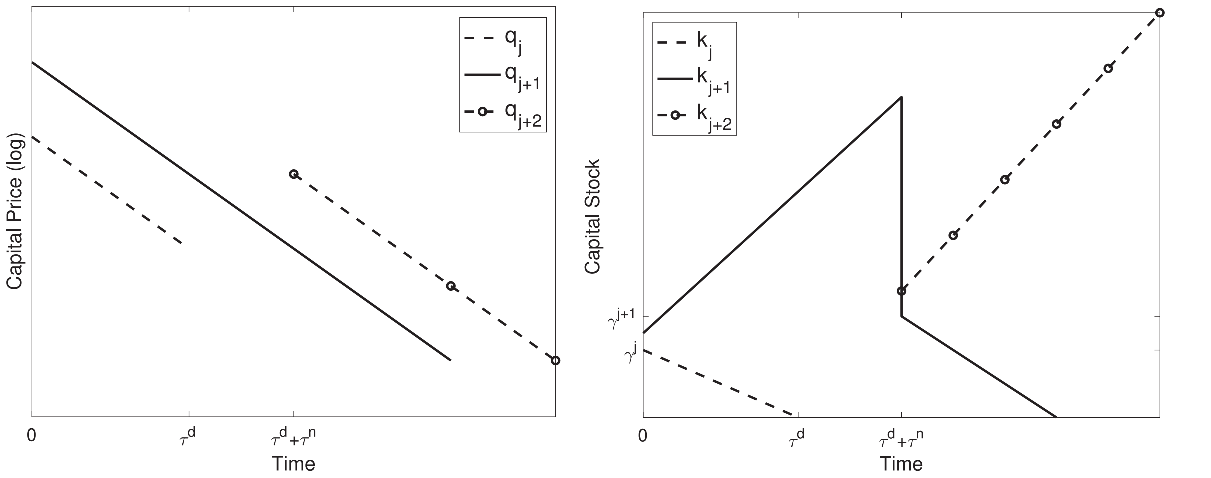
$$q_{j+2}(\tau^d) > v_{j+2}(\tau^d). \tag{12}$$

Therefore, it is not profitable to innovate and immediately adopt capital $j+2$ in the consumption sector.[11] Instead, it is still profitable to produce and accumulate capital $j+1$ in the investment sector. This continuing accumulation of machines of type $j+1$ decreases their price and, as a consequence, the implicit price of capital $j+2$ which, from above, is $q_{j+2} = q_{j+1} a$.

[9] Recall that the price for the final consumption good in units of the numeraire, i.e. marginal utility, is $1/c$. Hence the wage rate in units of the numeraire is w/c.

[10] The zero profit condition in the innovation sector is originally written in terms of capital prices. As shown in Appendix B.1, there is a linear relation between the price and the rental rate of capital. We show in B.3 that the innovation sector converting capital j' to j is activated when both are used in the consumption goods.

[11] Inequality (12) implies that the Tobin's Q for a new technology is smaller than 1 during the innovation phase and is equal to 1 during the adoption phase. Extending the model, for example, to incorporate adjustment costs associated with training of labor to use the new machines, would allow for a Tobin's Q larger than 1 in the adoption phase.



(a) Capital Price (log) (b) Capital Stock

Fig. 2.1. The dynamics of capital price (log) and stock of different vintages.
Note: This figure plots the evolution of capital price and stock of different vintages over two consecutive cycles of the model. A full growth cycle contains an adoption phase, from 0 to τ^d, and an innovation phase from τ^d to $\tau^d + \tau^n$.

This we call the "$j+1$ innovation phase", during which $j+1$ is the only capital employed in the consumption sector. It ends when q_{j+2} reaches its value (v_{j+2}) in the production of consumption. At that point, capital $j+2$ is invented and a new $(j+1, j+2)$ adoption phase starts.[12] It is shown in Appendix B.2 that, during the innovation phase, the implicit price of capital $j+2$ needs to decrease by a factor of a, at the rate of $b-\rho$. Simple algebra implies that the innovation phase lasts for $\tau^n = \frac{\log a}{b-\rho}$ units of time.

Fig. 2.1 illustrates the evolution of capital price (left) and capital stock (right) over two consecutive growth cycles in the model. Denote as $t=0$ the time when vintage $j+1$ is used for the first time to produce the consumption good that, until then, was entirely produced by capital j. In the $(j, j+1)$ adoption phase, capital j is converted to $j+1$ which accumulates over time, driving its price (as well as the price of capital j from the zero profit condition of the innovation sector) to decline. The adoption phase ends at time $t=\tau^d$, at which point capital j is phased out. In the following $j+1$ innovation phase from $t=\tau^d$ to $t=\tau^d+\tau^n$, capital $j+1$ keeps accumulating which further drives down its price. At $t=\tau^d+\tau^n$ when the innovation phase ends, the implicit price of capital $j+2$ is low enough and capital $j+2$ is invented and adopted in production, starting a new $(j+1, j+2)$ adoption phase. The price of capital $j+1$ keeps declining in the new $(j+1, j+2)$ adoption phase until it is fully phased out at the end of the phase.

Fig. 2.2 illustrates the allocation of labor (left) and the evolution of aggregate consumption output (right). In the $(j, j+1)$ adoption phase consumption increases over time as machines of vintage j are replaced by those of vintage $j+1$ and labor reallocates from the former to the latter. In the following innovation phase from $t=\tau^d$ to $t=\tau^d+\tau^n$ output of the consumption sector is constant as labor is already fully employed by capital $j+1$.

This implies the following corollary.

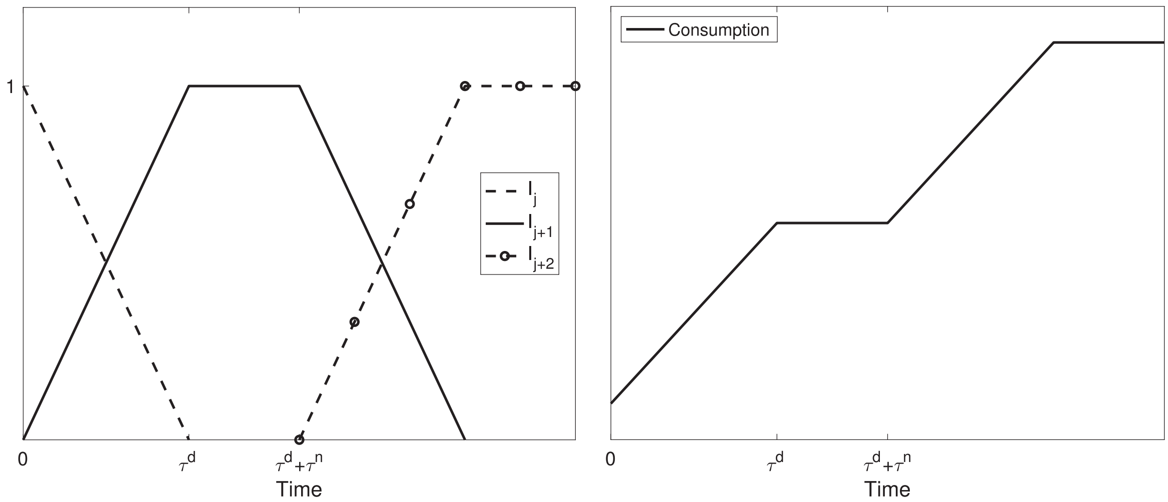
Corollary 1. *The labor share declines from* $\frac{a-1}{a-1/\gamma}$ *to* $\frac{1}{\gamma}\frac{a-1}{a-1/\gamma}$ *in an adoption phase. The labor share increases in the following innovation phase and goes back to* $\frac{a-1}{a-1/\gamma}$ *at its end. Further, in an adoption (innovation) phase, labor productivity increases (stagnates), and wages stagnate (increase)*

During the $(j, j+1)$ adoption phase, from Eq. (10), the wage rate is $w = \gamma^j \frac{a-1}{a-1/\gamma}$. The labor shares in firms employing technology j and $j+1$, respectively, are

$$LS_j = \frac{wl_j}{\gamma^j l_j} = \frac{a-1}{a-1/\gamma}, \qquad LS_{j+1} = \frac{wl_{j+1}}{\gamma^{j+1} l_{j+1}} = \frac{1}{\gamma}\frac{a-1}{a-1/\gamma}. \tag{13}$$

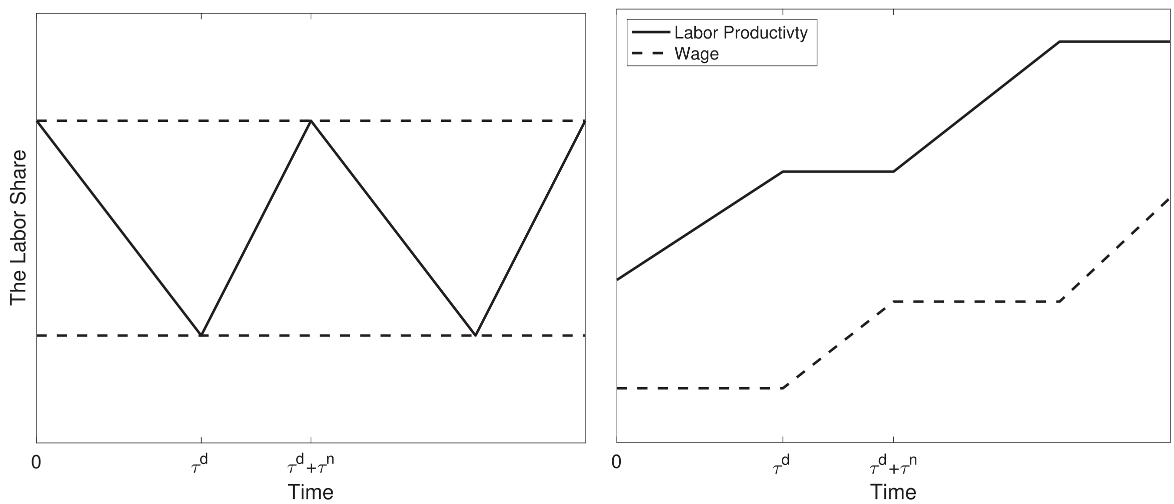
As $\gamma > 1$, $LS_{j+1} < LS_j$. Reallocation of labor from capital j to capital $j+1$ decreases the aggregate labor income share.

[12] A second, equivalent, path is the following. Invention of $j+2$ happens immediately and the new machine is created at the end of the adoption phase. It would still not be profitable to use it in the consumption sector, while it would satisfy the zero-profit condition to use the new machines to start the self-accumulation process in the investment sector. This continues until its price drops to make it profitable using capital $j+2$ in the consumption sector. One can think of this as a small innovative start-up that accumulates productive capacity and finances its temporary losses by borrowing against the promise of future revenues. Actually, this argument implies that the new technology can be created at any point in time during the "innovation" phase, the only difference being the "size" of the start-up firms and the length of time during which they self-accumulate productive capacity before selling it to the consumption sector. All such "different" paths are payoff-equivalent in the sense that the time at which vintage $j+2$ capital starts to be used in the consumption sector is the same and the same productive capacity is used. Consumption paths, and utilities, are therefore identical.



(a) Labor Allocation (b) Consumption

Fig. 2.2. Labor allocation and the dynamics of consumption.
Note: The left panel illustrates the evolution of labor allocation; the right panel reports the evolution of the aggregate consumption output.



(a) The Labor Share (b) Labor Productivity and Wage

Fig. 2.3. The dynamics of the aggregate labor share, labor productivity and wage.
Note: The left panel illustrates the evolution of the aggregate labor share; the right panel reports the dynamics of the aggregate labor productivity and wage.

In the following innovation phase, all labor is employed by technology $j+1$ in the consumption sector. The accumulation of additional productive capacity reduces the price of capital $j+1$ and its rate of return. From the zero profit condition in the consumption sector,

$$w/\gamma^{j+1} = 1 - r_{j+1}, \tag{14}$$

a decline in r_{j+1} increases the left-hand side, which is the labor share during the innovation phase. At its minimum, $r_{j+1} = \frac{1}{a}\frac{\gamma-1}{\gamma-1/a}$, implying that the labor share is again $\frac{a-1}{a-1/\gamma}$ when the new $(j+1, j+2)$ adoption phase starts. Hence the labor share fluctuates within a cycle and stabilizes across cycles, as illustrated in the left panel of Fig. 2.3.[13]

[13] The factor shares we focus upon here are those for the consumption sector, as by assumption the labor share is zero in the other two sectors. In Appendix B.4, we compute them for the whole economy and provide parameter restrictions under which the same cyclical pattern holds.

When technology $j+1$ is invented and adopted, reallocation of labor from the old technology j implies that accumulation in $j+1$ brings about more consumption, hence its rate of return in terms of consumption does not decline, and, from the zero profit condition in the consumption sector, wages do not increase. In the innovation phase, forward-looking agents keep accumulating capital $j+1$ as the latter will, eventually, pay off through the production of the new technology $j+2$. However, diminishing returns to accumulating $j+1$ set in when it employs all labor in the consumption sector. Given the Leontief production function and fixed labor supply, further accumulation of capital $j+1$ does not lead to any increase in consumption. Declines in the rate of return to capital $j+1$ further imply a rise in wages as firms operating technology $j+1$ still earn zero profit in the innovation phase.[14]

The adoption of a new technology $j+1$ leads to an increase of the real wage only late, during the innovation phase, while the wage remains constant in the adoption phase. However, over a full cycle, all the productivity gains that technological change brings about are completely transferred, eventually, in the growth of the average real wage. The right panel of Fig. 2.3 illustrates the dynamics of labor productivity and wage.[15]

2.2. Initial conditions and the uniqueness of equilibrium

We have characterized the growth cycles the model converges to but, in doing so, we have abstracted from the initial conditions, and, along with it, the uniqueness of equilibrium. This section first establishes the relation between k_{j+2} at the beginning of the $(j+1, j+2)$ adoption phase and k_{j+1} at the beginning of the $(j, j+1)$ adoption phase, then derives from this relation a condition the value of k_1 should satisfy. This allows us to prove the uniqueness of the initial allocation and the following equilibrium path.

Denote with $j=0$ the least advanced type of capital. The first recurring cycle starts when machines of type 0 and type 1 are simultaneously used in producing the consumption good. Denote with τ_{j+1} the beginning of the $(j, j+1)$ adoption phase, i.e. the first time capital $j+1$ is employed in producing the consumption good, and with $k_{j+1}(\tau_{j+1})$ the stock of capital $j+1$ at τ_{j+1}. We establish in Appendix B.3 that $k_{j+2}(\tau_{j+2})$ and $k_{j+1}(\tau_{j+1})$ satisfy the following linear relation,

$$\frac{k_{j+2}(\tau_{j+2})}{\gamma^{j+2}} = (a\gamma)^{\frac{\rho}{b-\rho}} \frac{k_{j+1}(\tau_{j+1})}{\gamma^{j+1}} - \Lambda, \tag{15}$$

with $\Lambda \equiv a^{\frac{\rho}{b-\rho}}(\gamma^{\frac{\rho}{b-\rho}} - 1)\frac{(a-1/\gamma)(b-\rho)}{\rho a(\gamma-1)} > 0$.

The term $\frac{k_j(\tau_j)}{\gamma^j}$ is the capital stock of vintage j normalized by output. From Eq. (15) there exists a unique steady state value for the normalized capital stock, denoted as k^*. As $(a\gamma)^{\frac{\rho}{b-\rho}} > 1$, the mapping in Eq. (15) is steeper than the 45-degree line, and the system is not stable. An initial value below the steady state leads to a negative capital stock, and any initial value above it results in an explosion, both violating the transversality condition.[16] Hence the condition $\frac{k_1(\tau_1)}{\gamma} = k^*$ must hold when the economy enters the first $(0,1)$ adoption phase at $t = \tau_1$.

To determine the endogenous starting time of the first cycle, τ_1, and the initial capital allocation, we begin with the case $0 < k_0(0) < 1$, i.e. the initial stock of capital 0 is not enough to employ all labor at $t=0$. During this initial phase, there is excess labor and no reason to innovate. The competitive equilibrium assigns a certain amount of capital 0 to sector 1, denoted as $k_0^1(0)$, and the rest, $k_0(0) - k_0^1(0)$, to sector 2. $k_0^1(t)$ and $c(t)$ grow over time until all labor is employed at time $t = \tau_0^d$, when consumption equals 1. The first innovation phase starts at this point: capital 0 is accumulated further and consumption remains constant. This phase ends at $t = \tau_1$ when the implicit price of capital 1 equals its value in production. At this time k^*a units of capital 0 are converted into k^* units of capital 1 and the first adoption phase starts. As shown in Appendix B.3, the condition that the amount of normalized capital of vintage 1, at time τ_1, equals the steady state value, i.e. $\frac{k_1(\tau_1)}{\gamma} = k^*$, uniquely determines the initial capital allocation.

The unique equilibrium we have just derived can then be used to determine the initial sectoral allocation for any initial level of capital $k_0(0)$ with $k_0(0) \geq 1$.[17] Proposition 3 summarizes these results.

Proposition 3. *There exists a unique equilibrium path. There might be an initial phase when the capital of vintage* 0 *is employed and accumulated to reach the threshold level* $\hat{k}_0$. *This occurs when* $k_0(0) < \hat{k}_0 \equiv a\gamma k^* + 1$, *where* k^* *is defined as*

$$k^* \equiv \frac{\Lambda}{(a\gamma)^{\frac{\rho}{b-\rho}} - 1}, \tag{16}$$

[14] A more technical interpretation is as follows. The price of capital $j+1$, $q_{j+1}(t)$ declines at the rate $b-\rho$ due to self-accumulation. Proposition 1 establishes that $q_{j+1}(t) = r_{j+1}(t)/[bc(t)]$. In the adoption phase when $c(t)$ grows at the rate $b-\rho$, the decline of q_{j+1} at the rate $b-\rho$ implies a constant interest rate $r_{j+1}(t)$. In the following innovation phase when the accumulation of capital $j+1$ does not generate any consumption growth, a decline in q_{j+1} translates into a decline in r_{j+1}. We show later that although introducing endogenous labor supply and general CES production functions bring about growth in consumption during the innovation phase, they do not change the qualitative features analyzed here.

[15] During the $(j, j+1)$ adoption phase, consumption output increases from γ^j to γ^{j+1} while labor is fully employed and constant at 1. labor productivity hence increases over time. Labor productivity is stagnant during the following innovation phase as all labor has already been employed by technology $j+1$.

[16] See Figure B.1 in Appendix B.3 for an illustration, which plots the normalized capital stock $k_{j+2}(\tau_{j+2})/\gamma^{j+2}$ as a function of $k_{j+1}(\tau_{j+1})/\gamma^{j+1}$.

[17] In particular, if $k_0(0) \in [1, k_1(\tau_1) * a + 1)$, then 1 unit of capital 0 is used in producing the consumption good and the remaining for self-accumulation. If $k_0(0) \in [k_{j+1}(\tau_{j+1}) * a^{j+1} + \gamma^j * a^j, k_{j+1}(\tau_{j+1} + \tau^d) * a^{j+1})$ for some $j \geq 1$, then $k_0(0)$ is immediately converted into both capital j and $j+1$ and the economy jumps to the corresponding adoption phase. Finally, if $k_0(0) \in [k_{j+1}(\tau_{j+1} + \tau^d) * a^{j+1}, k_{j+2}(\tau_{j+2}) * a^{j+2} + \gamma^{j+1} * a^{j+1})$ for some j, then a portion of $k_0(0)$ is converted into capital $j+1$ to produce γ^{j+1} units of the consumption good while the rest goes to self-accumulation and the economy starts from the corresponding innovation phase.

with $\Lambda \equiv a^{\frac{\rho}{b-\rho}}(\gamma^{\frac{\rho}{b-\rho}} - 1)\frac{(a-1/\gamma)(b-\rho)}{\rho a(\gamma-1)}$*. After that initial phase, the economy settles into a recurring, full employment, cycle of adoption and innovation phases. The amount of capital stock* j *when it is first employed in the consumption sector at* $t = \tau_j$ *satisfies* $k_j(\tau_j) = \gamma^j k^*$.

Proof. See Appendix B.3.

2.3. Endogenous labor supply

We now relax the assumption that labor supply is fixed at one and set the representative agent's preferences to

$$\int_0^\infty e^{-\rho t}[\log c(t) - \zeta \frac{\eta - 1}{\eta} \ell(t)^{\frac{\eta}{\eta-1}}]dt, \quad \zeta > 0,\ \eta > 1. \tag{17}$$

The first-order condition for working hours $\ell(t)$ is

$$w(t)/c(t) = \zeta \ell(t)^{\frac{1}{\eta-1}}. \tag{18}$$

As in the baseline model, in an adoption phase the wage, determined by zero profit conditions, is still constant, and consumption grows at the rate $b - \rho$. Hence employment shrinks at the rate $(\eta - 1)(b - \rho)$. A shrinking supply of labor implies that it takes less time to reallocate all labor from the old to the new technology: endogenous labor shortens the adoption phase.

In the baseline model, during the innovation phase the accumulation of k_{j+1} brings no growth in the consumption good because there is no additional labor to employ. With endogenous labor, consumption can grow if the supply of labor increases. From Eq. (18) the growth rates of employment and consumption are equal to $(\eta - 1)/\eta$ times the growth rate of wage. We establish in Appendix B.5 that consumption does grow now but at a rate, smaller than $b - \rho$, hence the rental rate decreases at a rate slower than in the baseline model. Interestingly we find that the innovation phase lasts longer, while the length of a full cycle remains unchanged. Formally, the following proposition for the extended model with endogenous labor supply holds

Proposition 4. *The economy with endogenous labor supply displays growth cycles, each consisting of an adoption phase when consumption grows at the rate* $b - \rho$*, and an innovation phase when consumption grows at the rate* $\frac{\frac{\eta-1}{\eta}\log\gamma}{\log a + \frac{\eta-1}{\eta}\log\gamma}(b-\rho)$*. The adoption phase lasts for* $\tilde{\tau}^g = \frac{\log\gamma}{\eta(b-\rho)}$*, and is followed by an innovation phase lasting* $\tilde{\tau}^n = \frac{\log a + \frac{\eta-1}{\eta}\log\gamma}{b-\rho}$*. The total length of a cycle remains*

$$\tilde{\tau}^* = \frac{\log a + \log\gamma}{b - \rho}.$$

Further, both the labor share and employment decline in the adoption phase while they increase during the innovation phase. In an adoption (innovation) phase, the labor productivity increases (stagnates), and the wage stagnates (increases).

Proof. See Appendix B.5.

3. Comparing the predictions of the model to the evidence

Price of, and return to capital. In the model, each capital vintage has a rise-then-fall life cycle. Over this cycle the price of the capital good, relative to the price of the consumption good, declines from $\frac{1}{b}\frac{\gamma-1}{\gamma-1/a}$ to $\frac{1}{ab}\frac{\gamma-1}{\gamma-1/a}$, generally consistent with the declining pattern in data (Gomme and Rupert, 2007). In addition to the fluctuations in the set of macro-variables considered so far, our model also implies fluctuations in the return to capital. During the $(j, j+1)$ adoption phase, the aggregate return to capital, a weighted average of return to vintages j and $j+1$, rises from $\frac{1}{a}\frac{\gamma-1}{\gamma-1/a}$ at the beginning to $\frac{\gamma-1}{\gamma-1/a}$ at the end of the phase. In the following innovation phase, it declines gradually back to $\frac{1}{a}\frac{\gamma-1}{\gamma-1/a}$, consistent with the return to capital in data that fluctuates within bounds.[18]

Cross correlations. With endogenous labor supply our model predicts that, during the adoption phase, labor share and employment decrease while labor productivity rises and the wage remains constant. During the innovation phase productivity is constant, while wage, employment, and labor share increase. These patterns imply the following correlations.

(i) The growth rates of labor share and labor productivity are negatively correlated within a growth cycle.
(ii) The growth rates of labor share and real wage are positively correlated.
(iii) The growth rate of the labor share is positively correlated with those of employment and hours worked.

To compare these theoretical correlations with those in the data, we focus on the non-financial corporate sector as our model has nothing to say about the public, financial, and self-employment sectors. A decomposition of the capital share shows that its

[18] Figures C.1 in Appendix C presents the relative price of investment to consumption goods in the model, and Figure C.2 shows the rate of return to capital in both model and data.

Table 3.1
Correlations between growth rates in LS and in other labor market variables.

Frequency	ΔLP	$\Delta WAGE$	ΔEMP	$\Delta HOUR$
Quarterly	-0.49^{***}	0.40^{***}	-0.29^{***}	-0.35^{***}
8-Quarter Moving Average	-0.40^{***}	0.38^{***}	0.21^{***}	0.09
12-Quarter Moving Average	-0.35^{***}	0.42^{***}	0.29^{***}	0.17^{***}
16-Quarter Moving Average	-0.23^{***}	0.48^{***}	0.37^{***}	0.25^{***}
H-P trend	-0.18^{***}	0.41^{**}	0.51^{***}	0.43^{***}

Note: For each variable, we first calculate the moving averages or the HP trend, and then the growth rate of the moving averages or H-P trend. *** : $p < 1\%$; ** : $p < 5\%$; * : $p < 10\%$. Data is for the Non-Financial Corporate Sector in the US, from 1947Q1–2023Q3.

dynamics is mainly driven by corporate profits.[19] We smooth out business cycle fluctuations by calculating moving averages (MA) over 8 quarters (2-year), 12 quarters (3-year), and 16 quarters (4-year), and by extracting the H-P trend with a smoothing parameter $\lambda = 1600$. Then we calculate the growth rate of the moving averages or of the H-P trend.[20]

Table 3.1 presents the correlation between the growth rates of the labor share (LS) and those of labor productivity (LP), wage rate (WAGE), employment (EMP), and working hours (HOUR) at different frequencies. At all frequencies, there is a significantly negative correlation between growth rates in LS and LP and a significantly positive correlation between growth rates in LS and growth rates in WAGE, consistent with the model predictions.[21] The correlation coefficient between the growth rate in LS and EMP/HOUR changes signs as we move from the business cycle to lower frequencies. The correlation is significantly negative for quarterly data but it becomes positive afterward, which suggests that the medium-run dynamics differ qualitatively from business cycle fluctuations, and is again consistent with the model.[22]

In Appendix C.3 we discuss the correlation between factor shares and output in both the model and the data, which we find important to better understand the implications of our theory. Section C.4 shows that the same correlation patterns as in Table 3.1 hold if the medium run components are extracted using a Baxter-King band-pass filter and also for the US Non-Farm Business Sector. In the same Appendix, we also split the data into *LS increasing* and *LS decreasing* periods, analogous to those in the model, and show that the average growth rates of the labor market variables within the two phases are consistent with the model predictions. Further, Appendix C.5 confirms that the labor share displays similar medium-run fluctuations and that the same correlations also hold for other OECD countries.

In addition to contemporaneous correlations, our model implies a specific pattern for dynamic correlations (lead–lags). A new technology that improves labor productivity, think of technology $j+1$ in the $(j, j+1)$ adoption phase, imposes an immediate negative impact on employment, wage, and labor share in the adoption phase. That impact, however, eventually turns positive in the following innovation phase. We provide a detailed analysis of the dynamic correlation patterns in data in Appendix C.6. Here we point out the one dimension along which our model fails to match the correlation in the data, that is the contemporaneous correlation, within a growth cycle, between the growth rates of labor productivity and wages. While labor productivity and wages are perfectly correlated in the model across the growth cycles – matching the strong positive correlation in the data – this is not the case within each cycle. In the latter case, when the labor productivity increases the wage is constant (adoption), while it grows when labor productivity is constant (innovation), which is not the case in the data where the positive correlation between LP and WAGE is always observed.

Last, in the model, the labor share declines during the adoption phase when labor is reallocated from less to more capital-intensive technologies. In Appendix C.7 we use data for the manufacturing sector from 1997 to 2007, a period with a significant decline in labor share, and exploit sector heterogeneity to show that the empirical correlations between reallocation and labor market variables are also consistent with the model.

4. Discussions and quantitative performances

4.1. CES production function

The Leontief production function in the consumption sector was assumed for analytical convenience. The qualitative features of the model are robust to using a more general constant-elasticity-of-substitution (CES) production function, with gross complementarity between capital and labor, in the consumption sector,

$$c_j = [k_j^{\frac{\sigma-1}{\sigma}} + (\gamma_j l_j)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, \quad \forall j, \quad 0 < \sigma < 1. \tag{19}$$

[19] Capital income is the sum of four different components: *rental income of persons*, *corporate profits*, *net interest and miscellaneous payments*, and *consumption of fixed capital*. Their shares in aggregate income are plotted in Figure C.3 in Appendix C. Corporate profits account for most of the cyclical pattern, while net interest is relatively acyclical, as is rental income.

[20] Growth rates are calculated using the difference in log except for the labor share, for which we simply take the first-order difference as it already is expressed in percentage terms.

[21] That the LS is defined as WAGE divided by LP does not necessarily imply a negative (positive) correlation between growth rates in LS and LP (WAGE). In Appendix Table C.1 we construct examples to show that, in principle, the correlations between the growth rates in LS and LP (or WAGE) may have either sign.

[22] Tables C.2 in Appendix confirm that correlations reported in Table 3.1 are robust to controlling for a linear trend and recession fixed effects.

We establish in Appendix D.1 that the economy still features recurring cycles with adoption and innovation phases. The length of the two phases becomes

$$\tau^d = (1-\sigma)\frac{\log\gamma}{b-\rho}, \quad \tau^n = \frac{\log a}{b-\rho} + \sigma\frac{\log\gamma}{b-\rho} \tag{20}$$

Hence the length of a full cycle remains equal to $\frac{\log a+\log\gamma}{b-\rho}$. In addition, during the $(j, j+1)$ adoption phase the labor share for technology j and $j+1$ is

$$LS_j = \frac{a^{1-\sigma}-1}{a^{1-\sigma}-(1/\gamma)^{1-\sigma}}, \quad LS_{j+1} = (1/\gamma)^{1-\sigma}\frac{a^{1-\sigma}-1}{a^{1-\sigma}-(1/\gamma)^{1-\sigma}} \tag{21}$$

The aggregate labor share declines from $\frac{a^{1-\sigma}-1}{a^{1-\sigma}-(1/\gamma)^{1-\sigma}}$ to $(1/\gamma)^{1-\sigma}\frac{a^{1-\sigma}-1}{a^{1-\sigma}-(1/\gamma)^{1-\sigma}}$ during the adoption phase, and climbs back to $\frac{a^{1-\sigma}-1}{a^{1-\sigma}-(1/\gamma)^{1-\sigma}}$ in the following innovation phase.

Under a CES production function, consumption still grows at the rate $b-\rho$ during the adoption phase. In the innovation phase capital accumulation brings about positive growth in consumption, and the basic intuitions driving our results remain intact. We show that the growth rate of consumption in an innovation phase is $g^n = \frac{\sigma\log\gamma}{\log a+\sigma\log\gamma}(b-\rho)$, which is strictly smaller than $b-\rho$. As in the baseline model, the rate of return on machines declines while the wage and the labor share increase during the innovation phase.[23]

4.2. Quantitative performance

The values of a and γ are independent of the capital vintage j by assumption. Allowing some degree of data-based heterogeneity in these parameters across vintages would generate cycles of different magnitudes as well as non-symmetric cycles. By varying the values of a and γ across vintages, our model could also generate a declining trend over multiple cycles.

For the H-P trend of the labor share in Fig. 1.1 the full decline-then-rising cycles in the post-WWII era contain: 1953Q2-1970Q4 (6.5 years), 1970Q4-1992Q1 (21.25 years), 1992Q1-2001Q1 (8 years), 2001Q1-2020Q1 (19 years). To give a sense of the magnitude of factor share fluctuations in the baseline model, we take the consumption growth rate, $b-\rho$, at 2% per year, recover the value of γ_j from the observed decline in the labor share, and set a to target a peak labor share level of two thirds over a full cycle. With parameter values calibrated this way, a decline of the labor share by 2–3 percentage points (pp) implies a length of 5–7 years for the whole cycle. A more pronounced decline of about 5 pp, as in the 2000–2014 period, leads to a cycle of more than 10 years. Using instead a CES production function with an elasticity of substitution between capital and labor set at $\sigma = 0.5$, close to the value estimated by Klump et al. (2007) and Oberfield and Raval (2021), a 5 pp decline in the labor share corresponds to a cycle of 21.86 years. These numbers align reasonably well with the data.

4.3. Other key parameters

The key parameter assumptions in our model are $b > \rho$, $a > 1$, and $\gamma > 1$. The key difference of our model from existing innovation models, e.g. Grossman and Helpman (1991), is that the productive capacity of new technology diffuses, possibly through imitation, gradually, represented by the parameter b. To see this point, note the inverse of the length of a full cycle is a good measure of the innovation intensity in our model,

$$\iota^* = 1/\tau^* = \frac{b-\rho}{\log a+\log\gamma} \tag{22}$$

As it becomes easier to reproduce knowledge capital, i.e. b is larger, the intensity of innovation increases. We construct in Appendix D.2 a version of the quality ladder model based on Grossman and Helpman (1991) that is comparable to ours. In particular, a still denotes innovation cost and γ the step size. The optimal innovation intensity in that economy is

$$\tilde{\iota} = \frac{1}{a} - \frac{\rho}{\log\gamma} \tag{23}$$

While the optimal innovation intensity in Eq. (23) is also affected by innovation cost, a, step size, γ, and discount rate, ρ, there is no role played by knowledge accumulation.[24] In a loose sense the Grossman–Helpman model, like all models in which technology/knowledge is a public good, assumes that $b = \infty$. As once a new technology is created an infinite copy of it can be produced. A finite number is actually produced only because of the monopoly power the innovator is awarded to deter imitation.

The accumulation process is fundamental in generating the factor share cycles in our model. Because $b < \infty$, after a new technology is invented, reallocating labor from the old to new technology takes time as the production capacity has to be built up gradually. Under $\gamma > 1$, new technology admits a lower labor share and labor reallocation reduces the aggregate labor share.

[23] The algebra for the CES case also clarifies why wage and interest rates stay constant during the adoption phase. In the $(j, j+1)$ adoption phase, capital of type $j+1$, and the labor it employs in the consumption sector, increase, while their ratio, hence their marginal productivities, remain constant. The same holds for technology j through the simultaneous decline of capital j and the labor it employs. We can also adopt a general CRRA utility, $u(c) = \frac{c^{1-\theta}}{1-\theta}$ without changing the main results. The price of consumption becomes $c^{-\theta}$, consumption grows at the rate $(b-\rho)/\theta$, and the length of a cycle becomes $\frac{log\gamma+1/\theta*\log a}{(b-\rho)/\theta}$.

[24] We explain in Appendix D.2 why step size, γ, affects the innovation intensity differently in the two models.

On the other hand, the assumption $a > 1$ is needed for the economy not to jump from one adoption phase immediately to the next. As $b > \rho$, the capital of a given vintage keeps self-replicating in the innovation phase, which brings about rises in wage and labor share.

The limit case: $a, \gamma \to 1$. Corollary 1 establishes that the labor share declines from $\frac{a-1}{a-1/\gamma}$ to $\frac{1}{\gamma}\frac{a-1}{a-1/\gamma}$ in an adoption phase, and increases back to $\frac{a-1}{a-1/\gamma}$ in the following innovation phase. It is straightforward to see that both a and γ are required to be strictly greater than 1 to generate endogenous factor share fluctuations. What will happen if both a and γ approach 1? To investigate the limit case, define $a = e^{\tilde{\alpha}\Delta}, \gamma = e^{\tilde{\gamma}\Delta}$, and let $\Delta \to 0$. It turns out that in the limit, the aggregate production function in the consumption sector converges to the following Cobb–Douglass function

$$\lim_{\Delta \to 0} F(K, L) = K^{\frac{\tilde{\gamma}}{\tilde{\alpha}+\tilde{\gamma}}} L^{\frac{\tilde{\alpha}}{\tilde{\alpha}+\tilde{\gamma}}}, \tag{24}$$

where K is aggregate capital in units of vintage 0 employed in the consumption sector, and L is the labor supply.[25] Hence the labor income share converges to a constant, $\frac{\tilde{\alpha}}{\tilde{\alpha}+\tilde{\gamma}}$.[26]

5. Conclusions

Since the end of WWII the factor shares of national income, in the US, and in other advanced market economies, have displayed relatively regular cycles. At the core of our theory is the idea that technical progress is labor-saving and responds to relative factor prices. Accumulation of capital embodying a given technology increases wages, which provides incentives for creating a new, labor-saving, technology embodied in new machines. This interaction between factor prices and labor-saving technical progress generates perpetual medium-run factor share cycles along the endogenous growth path.

Our model is stylized and deterministic, still, it is consistent with the main medium- and long-run facts. Our aim is to propose a mechanism capable of delivering both endogenous growth and factor share oscillations. Further work is needed to turn it into a quantitative model capable of replicating the detailed sample correlations of the main aggregate variables. The model focuses on deterministic medium-run dynamics abstracting from shocks and propagation mechanisms that are relevant at the business cycle frequencies. The business cycle implications of our theory, in the presence of some kind of random disturbances occurring at a quarterly frequency, are left for future research.

Data availability

Data will be made available on request.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmoneco.2024.103610.

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[25] Note when $\alpha \to 1$, the capital of different vintages becomes homogeneous. This result differs from but is closely related to the aggregation result in Jones (2005). See Appendix D.2 for details.

[26] In Appendix D.3 we further show that the baseline model can be extended to allow for exogenous population growth. There we also discuss the usage of labor in the investment/innovation sectors.

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