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Endowment, technology choice, and industrial upgrading ☆

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A B S T R A C T

This paper sets up a dynamic infinite-industry model to discuss the impact of factor endowments on technology choice and industrial upgrading. The model shows that at any time, the technologies in any industries should be adapted to the factor endowments structure. It further shows that as the economy develops, the industrial structure is unimodal with a leading industry, and with capital accumulation, each industry experiences technology upgrading from labor intensive to capital intensive and the leading industry shifts to more capital intensive as well. The model's implications are consistent with the stylized facts of manufacturing data from the United States and other countries. By incorporating various frictions and market failures, the model can discuss various policy issues in economic development.

1. Introduction

This paper studies technology choices and industrial upgrading in the industrial sector. The industrial sector, which refers to all manufacturing industries, is traditionally characterized as "the main engine of fast growth" (Kaldor, 1966, 1967; Murphy et al., 1989; Rodrik, 2009). Two patterns are observed in the industrial sector when the capital endowment becomes more abundant: the production technology in each disaggregated industry becomes more capital-intensive, at the same time, the more capital-intensive industries are gaining more market shares in the industrial sector. We aim to explain the observations by answering the following questions: how the aggregate endowment structure determines the technology choices in each industry and the aggregate industrial structure, and how the dynamics in endowment structure, technology choices, and industrial structural connect.

We first document the capital intensity and value-added share of each industry (hereafter, we refer to a manufacturing industry as an industry) in the economy, using the National Bureau of Economic Research — U.S. Census Bureau's Center for Economic Studies (NBER-CES) Manufacturing Industry Database from 1958 to 2011. We find a strong pattern that capital intensity (measured by the ratio of capital stock over employment or the capital income share in value added) in each industry, as well as in the whole industrial sector, is increasing over time. We also find that the distribution of market shares, measured by the value-added share of each industry in the total industrial sector, is unimodal. In addition, the leading industry, which has the largest market share in the industrial sector, shifts to more capital-intensive industries as the economy becomes more capital abundant. The pattern is persistent using the NBER-CES Manufacturing Industry Database and the China Industrial Productivity (CIP) data set, as well as the United Nations Industrial Development Organization (UNIDO) data set for cross-country analysis.

To capture the above empirical facts, we set up a dynamic infinite-industry general equilibrium model. A remarkable feature of our model is the consideration of technology choice, by which we mean a firm chooses a specific technology from a set of available technologies to conduct its production in a certain industry. Concretely, the technology choice is modeled as the choice of augmenting coefficients of capital and labor in a constant elasticity of substitution (CES) production function. The two augmenting coefficients are bounded by a technology frontier (an idea from Caselli and Coleman (2006)): technologies that are more efficient in using labor are less efficient in using capital, and vice versa. In the baseline model, the technology choice is costless. We model the cost of technology choice by reduction of the technology frontier in model extensions.

We have four key findings in the benchmark model. First, with a moderate discount rate, the aggregate economy grows unboundedly.

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Second, with the increase in the capital endowment, the capital rental — wage ratio declines, each industry experiences technology upgrading from labor intensive to capital intensive, and the capital intensity of each industry increases. Third, the technology choice in each industry depends on the aggregate endowments, technology frontier, as well as elasticity of substitution between capital and labor. When the elasticity of substitution between capital and labor is lower (higher) than one, the accumulation of the total capital endowment leads to lower (higher) capital-augmenting technology and higher (lower) labor-augmenting technology. Fourth, the industrial structure is unimodal, with a leading industry at any time in the process of economic development. The leading industry is the one for which the relative efficiency of capital over labor best matches the rental — wage ratio in equilibrium. Along with the increase in capital endowment, the industrial structure shifts more and more to capital-intensive industries, and the leading industry is taken over by a more capital-intensive industry continuously.

The model we build captures the first-best economy, where the changes in technology and industrial structure are costless. In practice, technology choice can incur costs. For example, the costs may arise from difficulties in acquiring patents, specific human capital for different technologies, and specific infrastructure investment to make the adoption feasible. We provide an extension of the model to incorporate the cost of technology choices. When the cost is low and is represented by some reduction in the output, the technical upgrading in each industry becomes slower. There will be a mismatch between industrial structure and factor endowment in the short-term and medium-term. In practice, the cost can be too high so that the technological progress and industrial upgrading come to a complete stop. Then there will be a need for government intervention, for instance, providing information, improvement in hard and soft infrastructures, and human capital, among others, in industrial upgrading. This paper thus provides a workhorse model for discussions on such market failures and the role of government intervention in industrialization.

Our paper is closely related to the literature on structural change, which studies the resource allocation across sectors, and matches the Kuznets facts, namely, the share of agriculture in gross domestic product (GDP) declines, the share of industry (manufacturing) exhibits a hump shape, and the share of services increases (Matsuyama, 2008; Herrendorf et al., 2014; Kongsamut et al., 2001; Caselli and Coleman, 2001; Wang and Xie, 2004; Acemoglu, 2009; Buera and Kaboski, 2012). Two main mechanisms of structural change are discussed in literature, that is, preference-driven and technology-driven. Our paper documents a similar pattern within the industrial sector that one leading industry exists at any specific time and the leading industry evolves during economic growth. The differences compared with the previous literature are that, first, we document the industrial upgrading of much more disaggregated industries within the industrial sector than in the structural change literature; second, we find that the change in factor endowments is a driving force for such industrial upgrading, and the leading industry is determined by the congruence of its capital intensity to the capital intensity of the whole economy. One recent paper, Ju et al. (2015), were the first to document the regular pattern of industrial dynamics in which each industry exhibits a bell-shaped life cycle, and a more capital-intensive industry reaches its peak later. Our model differs from that of Ju et al. (2015) in that in our model technology choice for any firm is endogenous, while in their model, technology is exogenously given so that they cannot address the change in technology choices.

This paper also adds on the literature on appropriate technology (Atkinson and Stiglitz, 1969; Diwan and Rodrik, 1991; Basu and Weil, 1998; Acemoglu and Zilibotti, 2001). The literature argues that countries with different endowments should choose different technologies. We are mostly connected to Caselli and Coleman (2006), who build a one-sector model of technology choices to study the imperfect substitution between skilled labor-augmenting and unskilled labor-augmenting technologies. The positive correlation between the skill premium and skilled labor endowment across countries is explained by imperfect substitution between unskilled and skilled labor, as well as differences in their technology frontiers. Other works on the appropriate technology include Chen (2020), Leonledesma and Satchi (2011, 2019), and Growiec (2013a) among others.

The existing literature on appropriate technology has dominatingly using one-sector models,[1] therefore cannot show the evolution of multi-industry industrial structure. In addition, the approach in Caselli and Coleman (2006) infers an outward shift of the technology frontiers along the dimension of the skilled labor–augmenting factor, since the return to skilled labor does not decrease with an increase in the supply of skilled labor. We contribute to the literature by constructing an infinite-industry model to relate the technical changes to the transformation of the industrial structure. In our framework, the expansion of skilled labor-intensive industries raises the demand for skilled labor and drives up the return to skilled labor. Therefore, the inferred changes in the technology frontiers would be smaller or not exist. Indeed, we provide an explanation for the general observations of capital deepening (Acemoglu and Guerrieri, 2008) and declining labor share (Karabarbounis and Neiman, 2014), by showing that the declining labor share in each industry could be due to the change in factor endowments and endogenous technology choices.

The rest of paper is organized as follows. In Section 2, we lay out two stylized facts on the observed capital intensity and market shares of disaggregated industries. Section 3 constructs the model and derives testable predictions. Section 4 computes technology choices based on the model, uses estimated results to simulate the model, and conducts counterfactual analysis of fixed technology frontiers. Section 5 concludes.

2. Stylized facts

In this section, we document two stylized facts on the capital intensity of disaggregated manufacturing industries, and the change in the distribution of their market shares in the process of an economy's development.

2.1. Capital intensity

In this subsection, we show that the capital intensities are heterogeneous across different industries and change over time.

We use the NBER-CES Manufacturing Industry Database, which covers 473 industries at the 6-digit North American Industry Classification System (NAICS) level from 1958 to 2011. The NBER-CES data provide detailed information on the total capital stock K, employment L, wage payment WL, and value added VA for each industry. Supplementary data from the UNIDO data set are also used, which cover 148 countries and 18 industries from 1963 to 2014. We compute the capital income share for each industry in each year using the NBER-CES data set, and that for each industry in each country, each year using the UNIDO data set.[2] We lay out the following findings.

Finding 1: There is tremendous cross-industry and over-time dispersion in capital shares.

[1] The literature on directed technology change, which emphasizes factor-biased technologies, has similarly taken the one-sector approach (Kennedy, 1964; Samuelson, 1965; Drandakis and Phelps, 1966; David, 1975; Acemoglu, 1998, 2002, 2007; Jones, 2005).

[2] Both the capital–labor ratio and capital income share (measured as one minus the share of wage income in value added) are often used to measure capital intensity. We find similar patterns when we replace capital income share with capital–labor share using the NBER-CES data set. The results are shown in Figures A1 in Appendix D. However, capital stock is not accessible in the UNIDO data set. Therefore, only capital income share is used to measure capital intensity.

Table 1
Within-industry changes in capital shares.

	(1) NBER-CES	(2)	(3) UNIDO	(4)	(5)
	cap_share	cap_labor ratio	cap_share	cap_share	cap_share
Time	0.00392***	0.0308***	0.000215***	0.000510***	0.000782***
	(186.08)	(253.69)	(4.63)	(11.3)	(19.47)
FE	Industry	Industry	No.	Country	Country*Industry
N	25,386	25,386	74,440	74,440	74,440
r2	0.837	0.902	0.000288	0.332	0.492

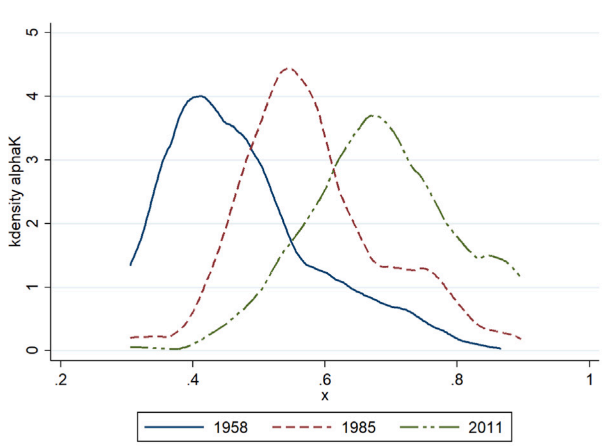

Fig. 1. Distribution of capital income shares.

Ju et al. (2015) document that there is large cross-industry heterogeneity in capital intensities. We also find large heterogeneity in capital shares across industries and over time. Fig. 1 shows the distribution of capital shares in 1958, 1980, and 2011 for U.S. industries. In 1958, the 90th percentile capital share was 0.65, which was 1.86 times the 10th percentile (0.35). In 2011, the 90th percentile capital share was 0.85, which was 1.55 times the 10th percentile (0.55). From 1958 to 2011, the average capital share evolved from 0.47 to 0.69, with persistent dispersion.

Finding 2: The increasing capital share is largely due to within-industry change.

After observing the above pattern, a natural question is whether the increasing capital share is due to within-industry changes or changes in industrial composition. We resolve this question by running an industry-year-level regression of capital intensity with the following equation:

$$CapitalIntensity_{it} = \beta_0' + \beta_1' t + \delta_i' + \epsilon_{it}',$$

where i denotes industry and t denotes time. After controlling for industry fixed effects, the coefficient β_1' shows the within-industry changes in capital intensity. To be more rigorous, we also conduct the following regression of capital share using the UNIDO data set, controlling for industry and country fixed effects:

$$CapitalShare_{cit} = \beta_0' + \beta_1' t + \delta_i' + \gamma_c' + \epsilon_{it}',$$

where c denotes country, i denotes industry, and t denotes time. After controlling for industry and country fixed effects, the coefficient β_1' indicates the within-industry changes in capital share. The regression results are shown in Table 1.

Column (1) shows the fixed effect regression using the NBER-CES data, where the capital intensity is measured by capital income share. The coefficient on t shows the average effect of the increasing capital share across industries. In column (2), capital intensity is measured by the capital–labor ratio. The estimates of β_1' are positive and significant in the two columns. Since we control for industry fixed effects, the coefficient indicates that the increasing capital share and capital intensity are due to within-industry changes. Similarly, we use the UNIDO data set to conduct a country-industry-year regression of capital share with respect to time. Column (3) shows the results for the ordinary least squares regression; in column (4), we add country fixed effects; and in column (5), we control for country-by-industry fixed effects. The increasing coefficient estimates of β_1' indicate that within-industry changes in capital share make up the main contribution to the aggregate changes.

We further decompose the change in average capital intensity into a within-industry term and a between-industry term. We follow the approach in Brandt et al. (2017), and shows that the change in weighted average capital intensity can be decomposed as[3]:

$$\Delta ks = \sum_i \overline{S}_i \times \Delta ks_i + \sum_i \Delta Si \times (\overline{ks}_i - ks_{i0}),$$

where i denotes each industry, and Δks is the change in a share weighted average of industry-level capital intensity. $\overline{S}_i$ is the average market share of industry i, Δks_i is the change in the capital intensity of industry i from 1958 to 2011. Similarly, $\overline{ks}_i$ is the average capital intensity of industry i, ΔSi is the change in the market share of industry i from 1958 to 2011. ks_{i0} is the capital intensity of industry i in 1958. The first term measures within-industry change, while the second term measures between-industry change. We find that both the within-industry and between-industry terms are positive. The within-industry term accounts for 83.66% of the total change, while the between-industry term accounts for 16.34%, i.e., the within-industry change in capital intensity makes up the main contribution to the aggregate change.

2.2. Market shares

In this subsection, we investigate the distribution of market shares of disaggregated industries in total industrial value added. We find that the distribution is unimodal, and the leading industry moves from labor-intensive to capital-intensive industries as the economy becomes more capital abundant. We show two categories of empirical findings. First, using separate data sets for two countries, the United States and China, we show similar patterns of change in the distribution of market shares for industries. Second, using the UNIDO data set, we conduct a panel data regression and show that the pattern we illustrate is universal.

Finding 1: The distribution of industries' market shares is unimodal in the United States and China.

We use two separate data sets to investigate the distribution of market shares of industries. The NBER-CES data set provides a long time series of production data for each industry in the United States. As

[3] Brandt et al. (2017) decompose the change in average industry productivity into four terms: within, between, entry and exit. Since the composition of industry in the NBER-CES dataset stay unchanged during 1958 to 2011, the entry and exit terms are both zero in our calculation.

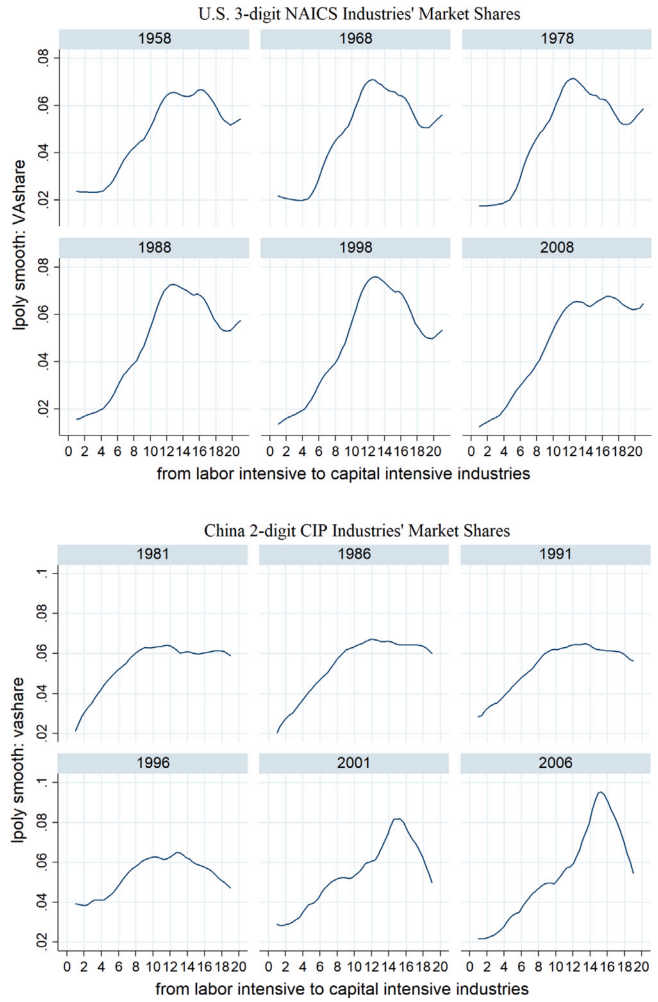

Fig. 2. Market shares of disaggregated industries in the United States and China.

a comparison, we use the CIP data, which cover 30 industries in China from 1981 to 2010.[4] We use the two data sets to compute the value-added shares of each 2-digit industry in each year, as well as the sample period average capital intensity of each industry. Then we show the relationship between market share and average capital intensity across industries. Fig. 2 shows the results.

The upper panel in Fig. 2 uses U.S. data and the lower panel uses Chinese data. The horizontal axis is the ranking of average capital intensity, from labor-intensive to capital-intensive industries. The vertical axis is each industry's value-added share in total industrial value added. The line is the local polynomial fit curve. The graphs shows that the distribution of market share is unimodal. In addition, the leading industry, which is captured by the peak of the distribution, moves towards more capital-intensive industries as the economy becomes more capital abundant.[5]

Finding 2: Cross-sectional analysis using the UNIDO data set shows that the distribution of industries' market shares is unimodal

Next, using the UNIDO data set, we show that the distribution of industries' market shares is unimodal. In addition, in more capital abundant countries, the leading industry is more capital intensive. We define the capital intensity of each industry as the sample-period average capital intensity of each industry in the United States. Then we rank the industries from labor intensive to capital intensive.

Table 2
Market shares and capital intensity.

	(1) $VAshare_{cjt}$	(2) $VAshare_{cjt}$	(3) $VAshare_{cjt}$	(4) $VAshare_{cjt}$
$Kshare_j$	5.228***	5.103***	5.268***	5.139***
	(83.14)	(81.96)	(80.27)	(78.31)
$Kshare_j^2$	−3.751***	−3.657***	−3.831***	−3.842***
	(−82.07)	(−80.81)	(−79.84)	(−80.02)
ω_{cjt}			0.0189***	0.0179***
			(35.61)	(33.67)
$Kshare_j \times Kabundance_{ct}$				**0.230*** **
				(16.39)
Country-year FE	No	Yes	Yes	Yes
N	82,937	82,937	75,616	75,549
R^2	0.0826	0.179	0.192	0.189
$\mathbb{F}$	**4009.57**	**3950.47**	**2846.39**	**2169.11**

The empirical specification is the following:

$$VAshare_{cjt} = \beta_1 Kshare_j + \beta_2 Kshare_j^2 + \beta_3 Kshare_j \times Kabundance_{ct} + \beta_4 \omega_{cjt} + \delta_{ct} + \varepsilon_{cjt}$$

where c, j, and t denote country, industry, and time, respectively. To reduce the endogeneity of one industry's capital intensity, we measure one industry's capital intensity by $Kshare_j$, the sample-period average capital income share of each industry in the United States. $Kabundance_{ct}$ denotes country-year-level capital abundance, measured by the total capital income share of each country c in time t. ω_{cjt} is labor productivity. δ_{ct} is country-year fixed effects. The standard errors are clustered at the country-industry-year level.

Table 2 presents the results. In column (1), we only include $Kshare_j$ and its quadratic term. It shows that $\beta_1 > 0$ and $\beta_2 < 0$, indicating that the distribution of market shares $VAshare_{cjt}$ over capital intensity has an inverse-V shape, with the leading industry's capital intensity measured by $-\beta_1/2\beta_2$. In column (2), we include country and year fixed effects. The results are qualitatively robust, suggesting that the pattern is persistent after controlling for country-specific features and macro shocks. In column (3), we further include labor productivity and the results are still robust.[6]

In column (4), we include an interaction term between each industry's capital intensity $Kshare_j$ and country-year-specific capital abundance $Kabundance_{ct}$. The coefficient on the interaction term is positive and significant. This finding suggests that, with an increase in $Kabundance_{ct}$, the capital intensity of the leading industry, $-(\beta_1 + \beta_3)/2\beta_2$, becomes larger, indicating that in a more capital abundant economy, the leading industry is more capital intensive.

3. Model

In this section, we set up a general equilibrium model with infinite industries. Under some general assumptions, the model produces results that are consistent with the empirical observations in Section 2.

3.1. Formal setup

Consider a completely competitive closed economy, which exists on the whole time interval $[0, \infty)$. There is a continuum of homogeneous individuals on $[0, 1]$ and each is alive on the whole time interval $[0, \infty)$,

[4] For an introduction to the CIP data set, see https://www.rieti.go.jp/en/database/CIP2015/index.html.

[5] To improve the visibility of the figure, we use the U.S. data for every 10 years, and Chinese data for every 5 years in Fig. 2. All years during the sample period are shown in Figure A2 in Appendix D.

[6] Since the data does not provide many control variables, the R^2 is limited. The $F-$ and $t-$ tests produce significant results, indicating that the endowment structure significantly determines the industrial structure.

each of which owns initial endowments of physical capital $K_0 > 0$ and labor $L_0 = 1$ at time $t = 0$, and no new individuals are born at any time $t > 0$. At any time, there is a continuum of industries on the interval $(0, 1)$: industry $i \in (0, 1)$ uses capital and labor to produce good i, an intermediate good; in addition, there is another sector that uses all the intermediate goods to produce the final good, which is the only consumption good and can be accumulated as capital.[7]

The life-long utility functional of the individual is

$$U = \int_0^\infty e^{-\varrho t} u(C(t))dt,$$

where $\varrho > 0$ is the individual's time discount rate; and $C(t)$ is his consumption of the final good at time t; and

$$u(C) = \frac{C^{1-\theta} - 1}{1 - \theta},$$

is the individual's instant utility function, where $\theta \in (0, 1]$.

For the final good sector, the production function is

$$Y = \left(\int_0^1 \theta_i Y_i^\rho \right)^{1/\rho},$$

where Y is the output of the final good, Y_i is the input of the intermediate good i, and $\rho \in (0, 1)$, $\theta_i > 0$, $\forall i \in (0, 1)$, and $\int_0^1 \theta_i di = 1$.

For any intermediate good sector $i \in (0, 1)$, there is a set of technologies

$$\mathcal{F}_i = \{ F_i^{(a_i, b_i)} | (a_i, b_i) \in \Theta_i \},$$

where $\Theta_i \subset \mathbb{R}_+^2 =: \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\}$, and

$$F_i^{(a_i,b_i)}(K, L) = \left((a_i K)^{\rho_i} + (b_i L)^{\rho_i} \right)^{1/\rho_i}, \quad \forall (K, L) \in \mathbb{R}_+^2, \tag{1}$$

where $\rho_i < 1$ is constant, K and L are the inputs of physical capital and labor, respectively, and, a_i and b_i are the augmenting coefficients for capital and labor. The firms in sector i can choose any one technology from $\mathcal{F}_i$, and, for simplicity, we here ignore the cost of transition from one technology to another.

We further assume

$$\Theta_i = \left\{ (a_i, b_i) \subset \mathbb{R}_+^2 : \left(\frac{a_i}{m_i} \right)^{\sigma_i} + \left(\frac{b_i}{n_i} \right)^{\sigma_i} = 1 \right\}, \tag{2}$$

where $m_i > 0$, $n_i > 0$, and σ_i are constants, satisfying $\sigma_i \rho_i > 0$. Here, m_i and n_i can be seen as the potential augmenting coefficients for capital and labor, respectively.

The set Θ_i can be called the technology frontier of industry i, as is done in Caselli and Coleman (2006).

The intuition behind assumption (1) is that in general, in each industry, we see in the real world that the elasticity between capital and labor is relatively stable, but the labor share is changing. So, for each industry, we make the posssible production functions CES functions with a fixed elasticity between capital and labor, and let the augmenting coefficients of capital and labor change.

The concrete form of the technology frontier (2) is motivated by an idea of Caselli and Coleman (2006), who use such a curve in their research on the substitution between skilled and unskilled labor. Our assumption (2) gains support from our empirical work in Section 4 (See also Section 3.2).

For simplicity, without any loss of generality, we assume that in each sector, there is only one firm. Moreover, we assume that throughout all the intermediate goods sectors, the depreciation rate of capital in the production processes is a common fixed constant and, for neatness of expression, we assume that this constant is 0.

In the above setting, it is only in intermediate goods sectors that the problem of technology choice exists. For the final good sector, the production function is given and fixed. In each intermediate good sector, the total production possibility set is the union of the production possibility sets for each possible technology, any one of which is convex, but their union is not. Hence, the classical Arrow–Debreu sufficiency conditions for the existence of Walrasian equilibrium are not satisfied fully, but here, we will see that the equilibrium exists and is unique.

For convenience, for any $i \in (0, 1)$, denote

$$\gamma_i =: \frac{m_i}{n_i}, \quad \tau_i =: \frac{\rho_i}{1 - \rho_i}, \quad \varepsilon_i =: \frac{\sigma_i \rho_i}{\sigma_i - \rho_i}, \quad \delta_i =: \frac{\sigma_i \tau_i}{\sigma_i - \tau_i},$$

and denote

$$r_* = \left[\int_0^1 \left(\theta_i m_i^\rho \right)^{\frac{1}{1-\rho}} \right]^{\frac{1-\rho}{\rho}}.$$

We make two technical assumptions:

$$\rho < \varepsilon_i < 1, \quad \forall i \in (0, 1), \tag{3}$$

$$\varrho < r_* < \frac{\varrho}{1 - \theta}. \tag{4}$$

Clearly, $\delta_i = \varepsilon_i / (1 - \varepsilon_i)$, and (3) is equivalent to $\varepsilon_i > \rho$, $\sigma_i > \tau_i$, $\forall i$.

Now, we provide an interpretation for the meaning of ε_i. Clearly, $\varepsilon_i^{-1} = \rho_i^{-1} - \sigma_i^{-1}$, then, the closer are ρ_i and σ_i, the larger is ε_i. The elasticity of substitution between K_i and L_i is $1/(1 - \rho_i)$ and, analogously, we can say that $1/(1 - \sigma_i)$ is the elasticity of substitution between a_i and b_i, or, in mathematics, ρ_i determines in some sense the shape of the production function isoquant curve for industry i, and analogously, σ_i determines in some sense the shape of the technology frontier for industry i. Then ε_i captures the joint features of the production function and the technology frontier of industry i. The larger ε_i is, the more similar are the shapes of the production function isoquant curve and the technology frontier of industry i. For simplicity, we call ε_i the degree of similitude in industry i, which roughly means the degree of coordination of the choice of the augmenting coefficients for capital and labor with the fundamental features of the capital and labor in industry i. In Section 3.2, we will see the economic meaning of ε_i more clearly.

The condition $\sigma_i \rho_i > 0$ means that σ_i and ρ_i have the same sign. That is, when capital and labor are complementary ($\rho_i < 0$, the elasticity of substitution between capital and labor is $1/(1 - \rho_i) > 1$), then, the augmenting coefficients for capital and labor a_i and b_i are also complementary; when capital and labor are substitutable ($\rho_i > 0$, the elasticity of substitution between capital and labor is $1/(1 - \rho_i) < 1$), then the augmenting coefficients for capital and labor a_i and b_i are also substitutable.

We now explain the meaning of (3). The condition $\rho < \varepsilon_i$ means that compared with each ε_i, ρ is relatively small, or equivalently, the elasticity of substitution between different goods $1/(1 - \rho)$ is relatively small. The condition $\varepsilon_i < 1$ guarantees that the optimal choice of a_i, b_i is an interior solution. The case of corner solution can be treated similarly and easier, hence, omitted.

As for (4), which is equivalent to $(1-\theta) r_* < \varrho < r_*$, we will see that r_* is the infimum of the equilibrium price of capital; thus, (4) means that the discount rate by which people discount the future should be located on the interval $((1 - \theta) r_*, r_*)$. If people discount the future too strongly such that the discount rate is greater than the interest rate, then the future will be relatively worthless, and, the economy will converge to a steady state and eventually stagnate. Instead, if people discount the future too weakly, then the technologies people own will make it possible that there will be infinitely many economic development paths enabling the people's life-long utility to be infinity, in which case, there will be no best economic development path. None of these scenarios coincides with reality. Hence, (4) is acceptable. In fact, (4) guarantees that the total capital will strictly increasing and goes to infinity finally, and the corresponding individual's life-long utility will be finite, just like in the classical Ramsey one-sector growth model with CRRA utility and linear technology.

[7] In this paper, we assume that for any variable depending on i, its trajectory along $i \in (0, 1)$ is piecewise smooth and right continuous.

Remark 1 (*Concerning the Production Functions*)**.** The above benchmark setting covers the Leontief case (see Appendix B). But this setting does not cover the Cobb–Douglas case, where for some (or all) i, $\rho_i = 0$. Thus, we will treat the Cobb–Douglas case in another work.

Throughout this paper, all the variables are nonnegative, and, for simplicity, in any optimization problem, we omit to write out the nonnegative constraints explicitly. Moreover, for neatness, for any dynamic variable, for example, $x(t)$, we write it simply as x, unless for clarity we need to emphases the time t; and for any path $x(t)$, we denote its derivative with respect to t as $\dot{x}$ (if it exists) and denote its growth rate as $\mathring{x} =: \dot{x}/x$, if $x > 0$, unless stated otherwise (the growth rate needs not to be constant, of course).

3.2. Local and global technologies

In this subsection, we discuss the technology set $\mathcal{F}_i$ in more detail, and give support for our assumption about technology frontier (2), and provide an interpretation of the economic meaning of ε_i.

For any intermediate sector $i \in (0,1)$, if we call any concrete $F_i^{(a_i,b_i)}$ a local technology, then,

$$F_i(K,L) = \max_{(a_i,b_i)\in\Theta_i} F_i^{(a_i,b_i)}(K,L), \quad \forall (K,L) \in \mathbb{R}_+^2$$

can be called the global technology for sector i.

Obviously, under the assumption that there is no technology transition cost, at any time, the firm in sector i always takes this global technology.

But we know that such a global technology is just an imaginary "combination", which has only symbolic meaning. The actual implementation of technology occurs only through concrete local technologies. We will show how the firms choose their technologies along the path of economic development and what determines their decisions.

Under the above assumptions about the technologies in sector i, we can obtain the global technology implicitly:

$$F_i(K,L) = \left((m_i K)^{\varepsilon_i} + (n_i L)^{\varepsilon_i}\right)^{1/\varepsilon_i}, \quad \forall (K,L) \in \mathbb{R}_+^2. \tag{5}$$

That is, the global technology is also of CES form; but there are changes in the elasticity of substitution between capital and labor and the augmenting coefficients of capital and labor.

We mention it again that this global technology exists only in the imaginary world. Although it seems like that the firm takes a CES technology with the elasticity of substitution between capital and labor $1/(1-\varepsilon_i)$, and the augmenting coefficients of capital and labor are m_i, n_i, respectively, this is not the phenomenon we can observe. What we observe is that at any time, the firm takes the CES technology with the elasticity of substitution between capital and labor $1/(1-\rho_i)$, and the augmenting coefficients of capital and labor are a_i, b_i, respectively. Under the above assumptions, we have $\varepsilon_i > \rho_i$, and hence, $1/(1-\varepsilon_i) > 1/(1-\rho_i)$, that is, the elasticity of substitution between capital and labor from the global view is larger than that from the local view.

As for the relationship between the parameter curve in (2) and the global technology in (5), we have the following statement, which we write as a lemma, which itself has some interest. We could use it in the proof of our main result, Theorem 1.

Lemma 1. [8] *Under the assumption (1), assertions (2) and (5) are equivalent.*

That is, under the assumption that any local technology is of CES form, as in (1), and the parameter, the augmenting coefficients of capital and labor, is located on some parameter curve, roughly, the global technology is of CES form if and only if the parameter curve is also of CES form.

This assertion gives some support to assumption (2), because if we want to maintain the property that the global technology still has a constant elasticity of substitution between capital and labor, then the parameter, the two augmenting coefficients, must be located on a CES curve. In other words, the elasticity of substitution between these two augmenting coefficients must be constant.[9]

At the end of this subsection, we return to the problem of the interpretation of the economic meaning of ε_i. In Section 3.1, mainly based on the geometric meaning, we call ε_i the degree of similitude of sector i. Now, it is clear that $1/(1-\varepsilon_i)$ is the elasticity of substitution between capital and labor under global technology F_i. Hence, we can call $1/(1-\varepsilon_i)$ the potential elasticity of substitution between capital and labor in sector i. Recall that under any concrete local technology $F_i^{(a_i,b_i)}$, due to the assumption that ρ_i may be positive or negative, capital and labor may be substitutable or complementary for each other. But under the global technology, capital and labor would always be substitutable for each other. Among all the intermediate industries, the greater the degree of similitude ε_i is, potentially, the stronger is the substitutability between capital and labor.

3.3. General equilibrium

In this subsection, we provide our formal definition of dynamic general equilibrium (equilibrium, for short). Let the price of the final consumption good be 1, that is, the final consumption good is the numeraire.

Definition 1. $\left(r^*(t), \omega^*(t), p_i^*(t), a_i^*(t), b_i^*(t), K_i^*(t), L_i^*(t), Y_i^*(t), Y^*(t), K^*(t),\right.$ $\left.C^*(t)\right)_{i\in(0,1),t\geq 0}$ is an equilibrium, if

(i) $(K^*, C^*) \in \arg\max_{(K,C)} \int_0^\infty e^{-\varrho t} u(C) dt$, s.t. $\dot{K} = r^* K + \omega^* - C$, $K(0) = K_0$;

(ii) for any t, $(Y_i^*(t))_{i\in(0,1)} \in \arg\max_{(Y_i)_{i\in(0,1)}} \left\{ \left(\int_0^1 \theta_i Y_i^\rho di\right)^{1/\rho} - \int_0^1 p_i^*(t) Y_i di \right\}$;

(iii) for any i,t, $(a_i^*(t), b_i^*(t), K_i^*(t), L_i^*(t)) \in \arg\max_{(a,b,K,L)} \{p_i^*(t)$ $((aK)^{\rho_i} + (bL)^{\rho_i})^{1/\rho_i}$ $-r^*(t)K - \omega^*(t)L\}$, s.t. $(a/m_i)^{\sigma_i} + (b/n_i)^{\sigma_i} = 1$;

(iv) for any i,t, $Y_i^*(t) = \left((a_i^*(t)K_i^*(t))^{\rho_i} + (b_i^*(t)L_i^*(t))^{\rho_i}\right)^{1/\rho_i}$; and for any t, $\int_0^1 K_i^*(t)di = Z^*(t)$, $\int_0^1 L_i^*(t)di = 1$, $Y^*(t) = \left(\int_0^1 \theta_i (Y_i^*(t))^\rho di\right)^{1/\rho}$.

We have the following result, which confirms the existence of the equilibrium and its uniqueness and presents a closed-form solution of the equilibrium.

Theorem 1. *For the above economy, there exists a unique equilibrium*

$$\left(r(t), \omega(t), p_i(t), a_i(t), b_i(t), K_i(t), L_i(t), Y_i(t), Y(t), K(t), C(t)\right)_{i\in(0,1),t\geq 0},$$

which is determined as follows: for any $t \geq 0$ and any $i \in (0,1)$,

$$Y = \left(\int_0^1 \theta_i Y_i^\rho di\right)^{1/\rho}, \quad Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i},$$

$$K_i = \frac{\left(\theta_i m_i^\rho (1 + z_i^{-1})^{\rho/\varepsilon_i - 1}\right)^{1/(1-\rho)}}{\int_0^1 \left(\theta_j m_j^\rho (1 + z_j^{-1})^{\rho/\varepsilon_j - 1}\right)^{1/(1-\rho)} dj} K,$$

[8] By use of the properties of envelope, the proof is easily obtained, hence, omitted. See also Growiec (2018). This lemma is essentially equivalent to Proposition 3 in Leonledesma and Satchi (2019).

[9] Growiec (2013b) provides a microfoundation for the formation of the technology frontier of a CES form as in (2), which can be derived as a contour line of the cumulative distribution function of the joint bivariate distribution of capital- and labor-augmenting ideas.

$$L_i = \frac{\left(\theta_i n_i^{\rho}(1+z_i)^{\rho/\varepsilon_i - 1}\right)^{1/(1-\rho)}}{\int_0^1 \left(\theta_j n_j^{\rho}(1+z_j)^{\rho/\varepsilon_j - 1}\right)^{1/(1-\rho)} dj},$$

$$a_i = m_i\left(1+z_i^{-1}\right)^{-1/\sigma_i}, \quad b_i = n_i\left(1+z_i\right)^{-1/\sigma_i},$$

$$z_i = \left(\gamma_i z\right)^{\delta_i}, \quad \omega = rz, \quad p_i = r\left((z^{-1}n_i)^{\delta_i} + m_i^{\delta_i}\right)^{-1/\delta_i},$$

$$r = \psi(z) =: \left(\int_0^1 \theta_i^{1/(1-\rho)}\left[(z^{-1}n_i)^{\delta_i} + m_i^{\delta_i}\right]^{\rho/[\delta_i(1-\rho)]} di\right)^{(1-\rho)/\rho},$$

where z is determined by[10]

$$K = \varphi(z) =: \frac{z^{1/(1-\rho)}\int_0^1 \left[\theta_i m_i^{\rho}\left(1+(\gamma_i z)^{-\delta_i}\right)^{\rho/\varepsilon_i - 1}\right]^{1/(1-\rho)} di}{\int_0^1 \left[\theta_j n_j^{\rho}\left(1+(\gamma_j z)^{\delta_j}\right)^{\rho/\varepsilon_j - 1}\right]^{1/(1-\rho)} dj},$$

and (K, C) is the unique solution of problem:

$$\max_{(K,C)} \int_0^{\infty} e^{-\varrho t} u(C) dt,$$
$$\text{s.t.} \quad \dot{K} = W(K) - C,$$
$$K(0) = K_0,$$

where $W(K) =: Kf(K) + g(K)$, and $f(K) =: \psi(\varphi^{-1}(K))$, $g(K) = f(K)\varphi^{-1}(K)$. And, it holds that $W'(K) = f(K)$ for any $K > 0$.

Remark 2 (*Concerning the Functions f, g, W*)**.** Clearly, f, g and W are all smooth, and f is strictly decreasing and g and W are strictly increasing. Further, $f(0) = \infty$, $f(\infty) = r_*$; $g(0) = 0$, $g(\infty) = \infty$; $W(0) = 0, W(\infty) = \infty$.

Geometrically, as a function of K, $W(K)$ is just the lower envelope of the family of lines $\{f(Z)K + g(Z) | Z > 0\}$, or, equivalently, the family of lines $\{f(Z)K + g(Z) | Z > 0\}$ is just the family of tangent lines of $W(K)$. Hence, in general, for any $K > 0$ and any $Z > 0$,

$$W(K) \le f(Z)K + g(Z).$$

Clearly, along the equilibrium path, $r = f(K)$, $\omega = g(K)$, and hence, $W(K) = rK + \omega$ is just the total income of the individual, and $W'(K) = r$, where $W'(K)$ and r are the shadow price and market price of capital, respectively. Thus, along the equilibrium path, the shadow price and market price of capital coincide exactly.

From Theorem 1, we see that all the variables $r, \omega, p_i, a_i, b_i, K_i, L_i, Y_i, Y$ and $\alpha_i, \beta_i, k_i, \eta_i, z_i$ are determined by K, and their relationships with K are homogeneous with respect to time. In other words, the relationships are independent of time.

At any time point, K represents the structure of the factor endowments, since the labor endowment is fixed and normalized to unit.

Therefore, we can state our first proposition as follows.

Proposition 1 (*The Decisiveness of the Structure of Factor Endowments*)**.** *At any stage of economic development, the structure of the factor endowments determines the technology choice, the price system, and the allocation of factors in all industries.*

In other words, at any time, the production arrangement should be adapted to the structure of the factor endowments, including the technology choice, distribution or allocation of factors among various industries, and the price system, including the prices of capital, labor, and various goods, is determined by the structure of the factor endowments.

[10] z is well-defined, which is due to the fact that the function φ in $z \in [0, \infty)$ is strictly increasing, taking values from 0 to ∞, accordingly. The function ψ on $z \in (0, \infty)$ is strictly decreasing, taking values from ∞ to r_*, accordingly.

3.4. Comparison of capital intensities among industries

Denote

$$r_i = \frac{r}{p_i}, \quad \omega_i = \frac{\omega}{p_i}, \quad k_i = \frac{K_i}{L_i}, \quad \eta_i = \frac{a_i}{b_i}, \quad \alpha_i = \frac{r_i K_i}{Y_i}, \quad \beta_i = \frac{\omega_i L_i}{Y_i}.$$

Here, k_i is the capital per capita in industry i; η_i can be considered a relative augmenting coefficient of capital to labor in industry i; α_i and β_i are the capital share and labor share, respectively, in industry i; r_i, ω_i are the relative price of capital and relative wage, respectively, relative to the price of good i; and z can be considered as the relative wage, relative to the price of capital. We will see below that z_i is just the ratio of capital share over labor share.

From Theorem 1, we obtain several corollaries.

Corollary 1. *For any $i, j \in (0, 1)$, $k_i > k_j \iff \alpha_i > \alpha_j$.*

Remark 3 (*Measurement of Capital Intensity*)**.** How should the capital intensity of an industry be measured? The ratio of capital over labor, or the capital income share? The latter is used in Ju et al. (2015) and in most of the other studies in the literature. In our setting, the two measurements are equivalent.

Corollary 2. *For any $i, j \in (0, 1)$, if $\varepsilon_i = \varepsilon_j$, then, $k_i > k_j \iff \gamma_i > \gamma_j$.*

That is, between firms with the same degree of similitude, γ_i and k_i are coherent as well.

Corollary 3. *For any $i, j \in (0, 1)$, if $\sigma_i = \sigma_j, \gamma_i = \gamma_j$, or $\rho_i = \rho_j$, then, $k_i > k_j \iff \eta_i\sigma_i > \eta_j\sigma_j$.*

Condition $\sigma_i = \sigma_j, \gamma_i = \gamma_j$ means that the technology frontiers for industries i and j are of the same type and have the same shape. The only difference between them is in their "sizes", or, to put it another way, roughly, one frontier is an enlarged version of the other frontier in both directions in the same magnitude. If the condition holds, then we say that industries i and j have similar technology frontiers.

Condition $\rho_i = \rho_j$ means that the elasticities of substitution between capital and labor in industry i and industry j are equal.

Corollary 3 says that for industries with similar technology frontiers or the same elasticity of substitution between capital and labor, if capital and labor are substitutable, then k_i and η_i are coherent as well; if capital and labor are complimentary, then the relationship between k_i and η_i is negative.

The intuition behind this is that when capital and labor are substitutable, an improvement in capital-augmenting technology would encourage firms to use more capital to substitute labor. By contrast, when capital and labor are complementary, if η_i is relatively larger, then the firm would use less capital to coordinate with more labor so that the effective capital and effective labor match each other.

3.5. Dynamics of (K, C)

In the above setting, the social planner's problem can be formulated as problem $\mathbb{P}$:

$$\max \int_0^{\infty} e^{-\varrho t} u(C) dt,$$
$$\text{s.t.} \quad \dot{K} = Y - C, \quad Y = \left(\int_0^1 \theta_i Y_i^{\rho} di\right)^{1/\rho},$$
$$Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i}, \quad \left(a_i/m_i\right)^{\sigma_i} + \left(b_i/n_i\right)^{\sigma_i} = 1, \quad \forall i,$$
$$\int_0^1 K_i di = K, \quad \int_0^1 L_i di = 1, \quad K(0) = K_0.$$

It can be verified that this problem has a unique solution that coincides with the equilibrium allocation. (As the proof is standard, we omit it.) Hence, the first and second theorems of welfare economics hold,

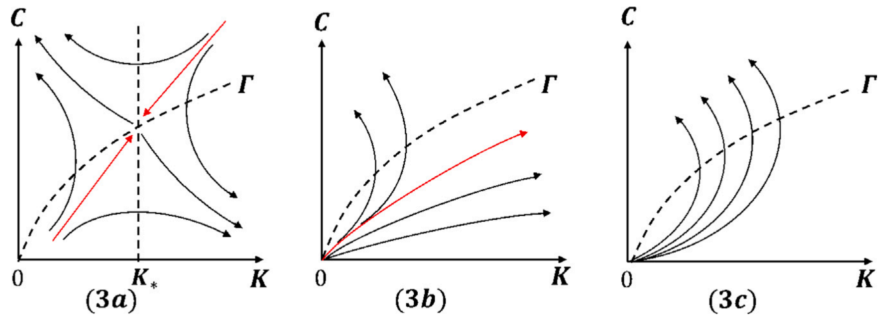

Fig. 3. Modes of economic development. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

that is, the equilibrium allocation is Pareto efficient, and any efficient allocation can be achieved by market competition. Hence, the dynamics of (K, C) from the market point of view is equivalent to the dynamics from the social planner's point of view.

It is easy to see that solving $\mathbb{P}$ is equivalent to simultaneously solving $\mathbb{P}_1$:

$$Y = W(K) = \max \left(\int_0^1 \theta_i Y_i^\rho di\right)^{1/\rho},$$
$$\text{s.t. } Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i}, \quad (a_i/m_i)^{\sigma_i} + (b_i/n_i)^{\sigma_i} = 1, \quad \forall i,$$
$$\int_0^1 K_i di = K, \quad \int_0^1 L_i di = 1,$$

and $\mathbb{P}_2$:

$$\max \int_0^\infty e^{-\varrho t} u(C) dt,$$
$$\text{s.t. } \dot{K} = W(K) - C, \tag{6}$$
$$K(0) = Z_0,$$

where W is defined as in Theorem 1.

Now we look at problem $\mathbb{P}_2$ in more detail. Clearly, the optimal path of economic development satisfies the Keynes–Ramsey rule:

$$\mathring{C} = \frac{\zeta - \varrho}{\theta}, \tag{7}$$

where $\zeta = W'(K) = f(K) = r$ is the shadow price (also the market price) of capital.

We know that along the equilibrium path, $W(K) = rK + \omega$, then, $\dot{r}K + \dot{\omega} = 0$. That is, along the equilibrium path, at any instant, an increase in the market value of labor is just offset by a decrease in the market value of capital, so that the net increase in the individual's wealth from the change in factor prices is zero.

We now return to the planar dynamical system (6), (7) of (K, C), which is delineate in a phase diagram in Fig. 3. Under the assumption (4), the phase diagram is (3b), in which the curve Γ is the separatrix $C = W(K)$, and the red trajectory is the optimal path, which goes to (∞, ∞). Any trajectory under the red one goes to (∞, ∞) as well, but its social welfare is less than that corresponding to the red trajectory. Any trajectory above the red trajectory crosses Γ and reaches the C-axis within a finite time and then the economy collapses.

Imagine the scenarios under different discount rates ϱ. Based on case (3b), along with ϱ getting larger and larger, the red trajectory will move upwards, after ϱ crosses the threshold r_*, that is, $\varrho > r_*$, the red trajectory will intersect Γ at a saddle. And the red trajectory itself will be divided into two parts, which are the two saddle paths, and the phase diagram will turn into (3a). In this case, all trajectories under the saddle paths will go the $(\infty, 0)$, and all trajectories above the saddle paths will touch the C-axis within a finite time and then the economy collapses.

In the other direction, based on case (3b), along with ϱ getting smaller and smaller, the red trajectory will move downwards. After ϱ crosses the threshold $(1-\theta)r_*$, that is, $\varrho < (1-\theta)r_*$, the red trajectory will "disappear" beneath the K-axis. Then all trajectories will cross Γ and touch the C-axis within a finite time and then the economy collapses. This is case (4c).

Compared with cases (3a) and (3b), case (3c) seems to be less obvious and needs to be justified. In fact, under the condition $\varrho < (1-\theta)r_*$, if the planar dynamical system (6), (7) has a trajectory such that $K \to \infty$ as $t \to \infty$, then, by $\dot{K} = f(K)(K+z) - C \le f(K)(K+z)$, and noticing $z = o(K)$ and $f(K) \to r_*$ as $K \to \infty$, we get that for large K, roughly,

$$\mathring{K} < r_* < \varsigma =: \frac{r_* - \varrho}{\theta} \sim \mathring{C},$$

then, again by $\dot{K} = f(K)(K+z) - C$, we get that for large K, $\dot{K} < 0$. A contradiction.

In (3a), $\varrho > r_*$, people discount the future too strongly. There is a unique, nontrivial saddle point (K_*, C_*) such that the solution of $\mathbb{P}_2$ is the saddle path converging to (K_*, C_*). Therefore, in this case, finally, the economy will converge to a steady state.

Of course, the saddle path is monotonic, not fluctuating. As usual, we can assume that the initial level of capital is small enough such that the capital stock will increase continuously, and so does the quantity of the final good. All the other variables in the economy will converge as well.

But convergence means stagnation. No stagnation nor collapse are suitable.

Hence, case (3b), which we obtain under assumption (4), is the only case that is consistent with the real-world scenario.

To sum up, under assumption (4) and from Theorem 1, we have the following corollary.

Corollary 4. *Along the equilibrium path, K, C, Y are all strictly increasing and converge to ∞.*

In short, the aggregate economy grows unboundedly.

3.6. Comparative static analysis with respect to K

We see that the economic variables, such as $r, \omega, p_i, a_i, b_i, K_i, L_i, Y_i, Y$ as well as α_i, β_i, k_i, are all determined by K, and their relationships with K are independent of time. Therefore, capital K is the key variable in the economy. From Section 3.5, we know that $K \to \infty$ monotonically along the equilibrium path as time goes to infinity.

Now, the question: how do these economic variables change along with the change in K?

Still from Theorem 1, we can derive the following corollary.

Corollary 5. *For any $i \in (0,1)$, with respect to K, $\alpha_i, k_i, \omega, \omega_i$ are strictly increasing; β_i, r, r_i are strictly decreasing; if $\rho_i > 0$, then, a_i is strictly increasing and b_i is strictly decreasing; and if $\rho_i < 0$, then, a_i is strictly decreasing and b_i is strictly increasing.*

Therefore, we have the following proposition.

Proposition 2 (*Technology Upgrading*). *With the economic development, each industry experiences technology upgrading from labor intensive to capital intensive, and accordingly, labor share is strictly decreasing.*

3.7. Dynamics of industrial structure

Now we consider the industrial structure and its changes along the equilibrium path. For any $i \in (0,1)$, denote

$$G_i = p_i Y_i, \quad w_i = \frac{G_i}{G}, \quad v_i = \frac{K_i}{K},$$

where $G = \int_0^1 G_i di$. Clearly, G_i is the value-added of industry i, G is the GDP, and w_i is the proportion of the value-added of industry i in the total GDP.

Then, $\{w_i\}_{i\in(0,1)}$, $\{v_i\}_{i\in(0,1)}$ and $\{L_i\}_{i\in(0,1)}$ are the distributions of value-added, of capital and of labor, respectively. In particular, the distribution of $\{w_i\}_{i\in(0,1)}$ can be used to express the industrial structure of this economy.

By the proof of Theorem 1 in Appendix C, we know that for any $i \in (0,1)$,

$$w_i = \frac{\left(\theta_i n_i^{\rho}(1+z_i)^{\rho/\delta_i}\right)^{1/(1-\rho)}}{\int_0^1 \left(\theta_j n_j^{\rho}(1+z_j)^{\rho/\delta_j}\right)^{1/(1-\rho)} dj},$$

then, for any $i, j \in (0,1)$,

$$\frac{w_i}{w_j} = \left(\frac{\theta_i n_i^{\rho}(1+z_i)^{\rho/\delta_i}}{\theta_j n_j^{\rho}(1+z_j)^{\rho/\delta_j}}\right)^{1/(1-\rho)}.$$

Clearly,

$$\frac{\partial}{\partial K}\left(\frac{w_i}{w_j}\right) = A_{ij}\left(k_i - k_j\right),$$

where $A_{ij} > 0$ is a function of K. Thus, $\frac{\partial}{\partial K}\left(\frac{w_i}{w_j}\right)$ has the same sign as $k_i - k_j$. Then, we have the following proposition.

Proposition 3 (*Structural Change*). *With the economic development, the industrial structure changes accordingly and tends more and more towards capital-intensive industries.*

In addition, clearly,

$$\lim_{K\to 0} w_i = \frac{(\theta_i n_i^{\rho})^{1/(1-\rho)}}{\int_0^1 (\theta_i n_i^{\rho})^{1/(1-\rho)} di}, \quad \lim_{K\to\infty} w_i = \frac{(\theta_i m_i^{\rho})^{1/(1-\rho)}}{\int_0^1 (\theta_i m_i^{\rho})^{1/(1-\rho)} di}.$$

In order to get more detailed results, we make more assumptions. First of all, without any loss of generality, we may assume that $\{\gamma_i\}_{i\in(0,1)}$ is strictly increasing, that is, the relative effectiveness of capital is strictly increasing from left to right. We further assume that for $\forall i \in (0,1)$, $\delta_i \equiv \delta$ for some constant $\delta > 0$.

Then, noticing that at any time point, for any $i \in (0,1)$, $k_i = z^{1+\delta}\gamma_i^{\delta}$, we have that k_i is strictly increasing with respect to i, that is, among the industries, from left to right, the capital intensities are strictly increasing. In other words, from left to right, the industries are becoming more and more capital intensive. And, clearly, such an ordering of $\{k_i\}_{i\in(0,1)}$ does not change in the whole process of economic development.

Therefore, in the stochastic order (on stochastic order, refer to Belzunce et al. (2016)), $\{w_i\}_{i\in(0,1)}$ is strictly increasing with the increase of K.

Further, since for any $0 < i < j < 1$,

$$\frac{v_i}{v_j} = \left[\frac{\theta_i m_i^{\rho}\left(1+(\gamma_i z)^{-\delta}\right)^{\rho/\sigma-1}}{\theta_j m_j^{\rho}\left(1+(\gamma_j z)^{-\delta}\right)^{\rho/\sigma-1}}\right]^{1/(1-\rho)}$$

and

$$\frac{L_i}{L_j} = \left[\frac{\theta_i n_i^{\rho}\left(1+(\gamma_i z)^{\delta}\right)^{\rho/\sigma-1}}{\theta_j n_j^{\rho}\left(1+(\gamma_j z)^{\delta}\right)^{\rho/\sigma-1}}\right]^{1/(1-\rho)}$$

are all strictly increasing respect to z and hence to K, then, $\{v_i\}_{i\in(0,1)}$ and $\{L_i\}_{i\in(0,1)}$ are all strictly decreasing in the stochastic order with the increase of K. This means that with economic development, labor flows from capital-intensive industries to labor-intensive industries, and capital, in distribution, also flows from capital-intensive industries to labor-intensive industries.

Under further more additional assumptions, we can obtain more detailed results on the patterns of $\{w_i\}_{i\in(0,1)}$, $\{v_i\}_{i\in(0,1)}$ and $\{L_i\}_{i\in(0,1)}$. More precisely, to give an example, we make the following additional assumption:

(A). For $\forall i \in (0,1)$, $\delta_i \equiv \delta$ for some constant $\delta \in (0,1)$, and as smooth functions of $i \in (0,1)$, $M_i =: \theta_i^{1/\rho} m_i$ is strictly increasing and strictly concave, $N_i =: \theta_i^{1/\rho} n_i$ is strictly decreasing and strictly concave, and $\lim_{i\to 0}\gamma_i = 0$, $\lim_{i\to 1}\gamma_i = \infty$.[11]

For simplicity of notation, for any positive variable x_i, as a smooth function of i, we use $\dot{x}_i = dx_i/di$ to represent its derivative with respect to i, and $\mathring{x}_i = \dot{x}_i/x_i$ to represent its growth rate with respect to i.

It is easy to verify that under **(A)**, as functions of i, $\{v_i\}_{i\in(0,1)}$ is strictly increasing, $\{L_i\}_{i\in(0,1)}$ is strictly decreasing, and $\{w_i\}_{i\in(0,1)}$ is unimodal, the mode of which is determined by

$$z = h_i, \tag{8}$$

where

$$h_i =: (\gamma_i^{1-\delta} H_i)^{1/\delta}, \quad H_i =: -\frac{\dot{N}_i}{\dot{M}_i},$$

and along with the economic development, $\{v_i\}_{i\in(0,1)}$ is getting more and more gentle, $\{L_i\}_{i\in(0,1)}$ is getting steeper and steeper, and the mode of $\{w_i\}_{i\in(0,1)}$ is moving towards right.

We call the industry corresponding to the mode of $\{w_i\}_{i\in(0,1)}$ the leading industry of the economy.

Now, we give interpretation for the economic meaning of (8). We call M_i the modified (by individuals' tastes) potential capital effectiveness of industry i, and N_i the modified (by individuals' tastes) potential labor effectiveness of industry i. Then, H_i is the marginal rate of substitution of modified potential capital effectiveness for modified potential labor effectiveness. That is, comparing neighboring industries, potentially, augmenting one unit of capital is equivalent to augmenting H_i units of labor. Hence, H_i is a rough measure of the modified (by individuals' tastes) strength of augmenting capital, relative to augmenting labor. Thus, H_i is strongly affected by individuals' tastes, but not the potential technologies only. Clearly, $H_i > 0$ is strictly increasing with respect to i. Recall that γ_i represents the potential relative capital effectiveness of industry i. And then, h_i measures the compromise of augmenting capital and augmenting labor, modified by individuals' tastes. We can call h_i the total relative capital effectiveness. Therefore, (8) can be interpreted as "the relative price of capital equals the total relative capital effectiveness".

Clearly, (8) is equivalent to

$$z_i = e_i, \tag{9}$$

where

$$e_i = -\frac{\mathring{N}_i}{\mathring{M}_i}$$

is the elasticity of substitution of M_i for N_i in industry i. Recall that z_i is the ratio of capital share over labor share of industry i, or the relative

[11] For example: for any $i \in (0,1)$, $\theta_i = (1-i)^{2\alpha\rho}/(2\alpha\rho+1)$, $m_i = Ai^{\alpha}(1-i)^{-2\alpha}$, $n_i = B(1-i)^{-\alpha}$, where $\alpha \in (0,1)$ and $A > 0, B > 0$ are constants.

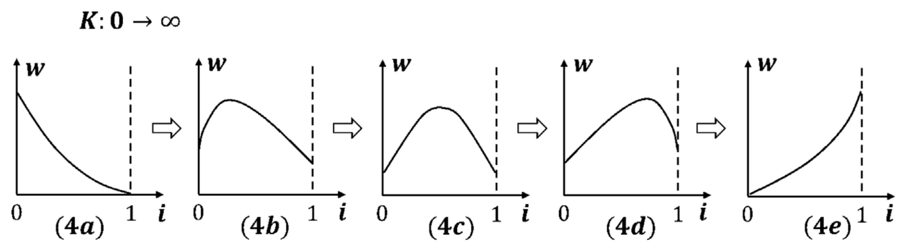

Fig. 4. Change in industrial structure.

capital share of industry i, for short. Then, (9) can be interpreted as "the relative capital share equals the elasticity of substitution of modified potential capital effectiveness for modified potential labor effectiveness".

From the above analysis, we see the pattern of the change in $\{w_i\}_{i\in(0,1)}$. At first (in the limiting extreme case, where $K = 0$), $\{w_i\}_{i\in(0,1)}$ is strictly decreasing with respect to i, as shown in Fig. 4 by (4a). Along with the increase in K, in the process of economic development, the old leading industry shrinks and the new leading industry shifts continuously rightward, as demonstrated in Fig. 4 by (4b), (4c), (4d), and finally (4e), which is another limiting extreme case, where $K = \infty$, $\{w_i\}_{i\in(0,1)}$ is strictly increasing with respect to i.

The mechanism of technology choice and structural change is as follows. Induced by the invisible hand, people allocate their income over all the commodities according to certain proportions, which depend on the elasticity of substitution between these commodities, and firms choose appropriate technologies and combinations of factors to maximize their profits. At any level of capital endowment, the market determines the relative price of capital over labor ($z = \varphi^{-1}(K)$) and other prices. Under this given price system, every firm organizes its production, including the choice of technology, so that the levels of capital per capita of all the industries are proportional to each other in fixed proportions. The leading industry appears, induced by market forces, according to the principle that its total relative capital effectiveness most closely matches the capital rental–wage ratio. This matching combines the two effects: relative capital effectiveness and individuals' preferences. Individuals prefer the commodities of some industries. On the one hand, firms tend to allocate more capital and labor to these industries to meet the demands of the individuals. On the other hand, subject to limited capital, if the relative levels of capital effectiveness of these industries are not very high, then the firms may not necessarily allocate more to these industries. Taking these two effects into account, firms choose the most suitable industry to which to allocate the most capital and labor, so that it becomes the dominant industry in the economy.

To sum up, in a world under assumption **(A)**, we have the following proposition.

Proposition 4 (*Pattern of Technology Upgrading and Structural Change*)**.** *At any stage of economic development, the industrial structure is unimodal with a leading industry. As the society's capital per capita grows, all industries simultaneously upgrade to more capital-intensive technology. The leading industry is continuously taken over by another new leading industry with greater capital intensity.*

3.8. Life cycle of any one industry

Any one intermediate industry has its own "life cycle". In general, it has a rising period, a peak period, and a declining period.

In the above economy under assumption (**A**), with economic development, each industry will be the leading industry and get to its peak at some time, which can be called the peak time of this industry. Before its peak time, it is in its rising period, that is, its share of GDP increases. After its peak time, it is in its declining period, that is, its share of GDP decreases and finally, converges to a specific level. The peak times of all the industries arrive sequentially: first, industries with lower potential capital intensities, followed by industries with higher potential capital intensities, one by one. This scenario is depicted in Fig. 5.

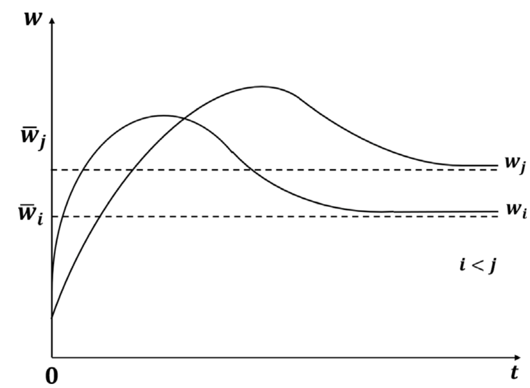

Fig. 5. Life cycle of two industries.

This aspect is also different from that in Ju et al. (2015), in which any industry has a limited life cycle, before which it has not yet set off, and after which it dies out.

3.9. Long-run state of the economy

We now turn to investigate the long-run state of this economy. That is, we investigate the asymptotic state of the economy, when t is large and hence K is large, and see if it finally goes along an asymptotic BGP.

Since $Y = W(K) = rK + \omega$ is the total output, or the GDP, then,

$$\alpha =: \frac{rK}{Y}, \qquad \beta =: \frac{\omega}{Y}, \qquad \vartheta =: \frac{K}{Y}$$

are the capital share, labor share and the capital–output ratio in the aggregate economy, respectively.

From Theorem 1, we can get

Corollary 6. *As $K \to \infty$, $r \to r_*$; $\omega \to \infty$; $\alpha \to 1$, $\beta \to 0$, $\vartheta \to 1/r_*$; and for any $i \in (0,1)$, $p_i \to r_*/m_i$; $a_i \to m_i$; $b_i \to 0$ or ∞, if $\sigma_i > 0$ or < 0; $\alpha_i \to 1$, $\beta_i \to 0$;*

$$v_i \to \frac{(\theta_i m_i^{\rho})^{1/(1-\rho)}}{\int_0^1 (\theta_i m_i^{\rho})^{1/(1-\rho)} di};$$

and for any $i, j \in (0,1)$,

$$\frac{L_i}{L_j} \to \left(\frac{\theta_i m_i^{\rho}}{\theta_j m_j^{\rho}}\right)^{1/(1-\rho)} \frac{\gamma_i^{\delta_i}}{\gamma_j^{\delta_j}} \quad or \quad 0, \quad if \quad \delta_i = \delta_j \quad or \quad \delta_i < \delta_j.$$

Therefore, we can give a proposition as follows.

Proposition 5 (*Long-Run State of the Economy*)**.** *In the long-run, asymptotically, the return of capital and the price of each intermediate good keep stable, the wage tends to grow unboundedly; and the labor share tends to 0* *exponentially, both in each industry and in the aggregate economy; and the capital–output ratio in the aggregate economy keeps stable; the distributions of capital and labor are stable; the technology in each industry almost purely relies on capital.*

By the way, it is easy to verify that for any $i \in (0,1)$, asymptotically,

$$Y_i \sim m_i K_i,$$

$$\mathring{Y}_i \sim \mathring{K}_i \sim \mathring{Y} \sim \mathring{K} \sim \mathring{C} \sim \varsigma.$$

And, by $K = \varphi(z)$, we get that asymptotically, $\mathring{z} \sim (1-\rho)\mathring{K}$, therefore,

$$\mathring{\beta} \sim -\rho\varsigma.$$

This indicates that in the long-run, the economy is almost a linear economy, and the equilibrium path is an asymptotic BGP. That is, after experiencing the "technology upgrading" and "structural change" in the early stage of development, at last, the economy goes on a BGP with the highest technologies. And, the labor share in the aggregate economy tends to 0 asymptotically by the rate of $\rho\varsigma$.

Remark 4 (*Differences Between Our Model and Those Of Ju et al. (2015)*). The key point of difference is that our model is a model about the technology choice and upgrading in a multi-sector economy. In our model, each industry experiences a process of technology upgrading from labor intensive to capital intensive as the economy develops. In Ju et al. (2015), technology is fixed for any industry.

On the mechanism for structural change, our explanation is more explicit: structural change is induced by technology upgrading, which, in turn, is driven by the change in factor endowment. Ju et al. (2015) presents two models, one with linear technology and the other with CES technology in the production of the final good. Mainly due to the complete substitution between the intermediate goods in the first model, at any stage of economic development, the industrial structure is like this: two and only two adjacent industries survive; all other industries disappear, so that the industrial structure is bell-shaped; and as the economy develops, the surviving industries become more and more capital intensive. In their second model, the industrial structure is not bell-shaped but strictly decreasing, that is, the more capital intensive an industry is, the less weight it has in the whole economy, and the slope of the industrial structure curve becomes more and more gentle as the economy develops. In our model, at any stage of economic development, all industries exist; the industrial structure is unimodal with a leading industry, and hence, bell-shaped; and the leading industry keeps changing to a more capital-intensive one continuously.

3.10. Modified model

We make two modifications to the above basic model.

3.10.1. Technical progress

In the above basic model, the technology frontier for any industry is fixed, and assumption (4) is crucial. However, without the assumption (4), we can obtain a similar result by allowing the technology frontier to expand outwards randomly, exogenously or endogenously.

One simple treatment is like this: instead of (2), we assume that at any time $t \geq 0$ and for any industry $i \in (0,1)$, the corresponding technology frontier is

$$\Theta_i(t) = \left\{ (a_i, b_i) \subset \mathbb{R}_+^2 : \left(\frac{a_i}{m_i}\right)^{\sigma_i} + \left(\frac{b_i}{n_i}\right)^{\sigma_i} = A(t) \right\},$$

where m_i, n_i, σ_i are constants as in (2), and $\{A(t)\}_{t\geq 0}$ is an increasing jump process, for example, a compound Poisson process from some initial state $A_0 > 0$, more precisely, for any $t \geq 0$,

$$A(t) = A_0 + \sum_{n=1}^{N(t)} \xi_n,$$

where $\{N(t)\}_{t\geq 0}$ is the standard Poisson process with parameter $\lambda > 0$, and $\{\xi_n\}_{n=1,2,\ldots}$ is a sequence of independent and identically distributed positive random variables with $E\xi_n = a \in (0,\infty)$, and $\{N(t)\}_{t\geq 0}$ and $\{\xi_n\}_{n=1,2,\ldots}$ are independent.

Then, the corresponding global production function for sector i will be

$$F_i(K,L) = A(t)\left((m_i K)^{\epsilon_i} + (n_i L)^{\epsilon_i}\right)^{1/\epsilon_i}, \quad \forall (K,L) \in \mathbb{R}_+^2.$$

That is, we assume that the technology frontiers are expanded outwards in the same pace for all industries, where A can be seen as the TFP in some sense, which is based on the stock of all ideas of humankind in production, since idea occurs randomly, and hence, we think that A should be represented by an increasing jump process. In addition, we assume that the instant utility function is $u(C) = \ln C$, that is, $\theta = 1$.[12]

It is easy to see that the equilibrium exists and is unique as well. In this case, $\mathbb{P}_2$ will turn to the form

$$\begin{aligned} &\max\ E\int_0^\infty e^{-\rho t}\ln C\,dt, \\ &\text{s.t.}\ \ dK = (AW(K) - C)\,dt, \\ &\qquad dA = d\sum_{n=1}^{N} \upsilon_n, \\ &\qquad K(0) = K_0, A(0) = A_0, \end{aligned}$$

where W is the same as in Theorem 1. Clearly, its optimal path satisfies that K is increasing and converges to ∞ almost surely, so does C. And hence, along the equilibrium path, almost surely, the technologies upgrade, and the economic structure changes, as in the above basic model.

In addition, denote the value function of $\mathbb{P}_2$ as $V(K,A)$, by the corresponding HJB equation, we can get the modified Keynes–Ramsey rule:

$$\mathring{C} = (Af(K) - \rho) + \lambda\left(\frac{EV_K(K, A+\upsilon_1)}{V_K(K,A)} - 1\right)$$

almost surely, where $f = W'$ as in Theorem 1.

Furthermore, for the limit behavior of the price system, we notice that in this modified model, in the long run, the price system for the commodities converges almost surely, as in the basic model, but, as to the prices of capital and labor, we see that they are all divergent to infinity almost surely, in particular, the price of capital, in the long run, will grow to infinity. In fact, in this modified model, we have

$$r = Af(K), \quad \omega = Ag(K),$$

where f, g are the same as in Theorem 1.

An interpretation is as follows. Since the technology for the production of the final consumption good does not change, and hence, in equilibrium, the price system for the commodities will converge, and in the meantime, since the technologies for the production of the intermediate goods are upgrading continuously and unboundedly, that is, for any $i \in (0,1)$,

$$a_i = Am_i\left(1 + z_i^{-1}\right)^{-1/\sigma_i}, \quad b_i = An_i\left(1 + z_i\right)^{-1/\sigma_i},$$

then, the prices of factors are all increasing unboundedly.

In contrast, in the basic model, in fact, in the long run, the technology upgrading stops at the highest technology $a_i = m_i$, and $b_i = 0$ or ∞, corresponding to $\sigma_i > 0$ or < 0, respectively.

We leave the consideration of endogenous R&D for further work.

[12] In this modified setting, if $\theta \in (0,1)$, then, the objective functional could be infinite.

3.10.2. Cost of technology choice

In reality, technology choice is not costless. Particular costs arise from patents or from the need of corresponding infrastructure and even the macroeconomic environment and policies.

For simplicity, here, we suppose that in each industry, the cost of technology choice is presented as the reduction of the output. More precisely, for sector i, if the inputs are K and L, then, the net output, net of the cost of technology choice, is

$$Y_i = \lambda_i \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i},$$

where $\lambda_i \in (0,1)$, denoting the effect of technology choice cost, the more the cost is, the less this λ_i is. And, this λ_i, in general, may change with time going on.

In this case, we may rewrite the net output as

$$Y_i = \left((\lambda_i a_i K_i)^{\rho_i} + (\lambda_i b_i L_i)^{\rho_i}\right)^{1/\rho_i},$$

and we see that $\lambda_i a_i$ and $\lambda_i b_i$ satisfy

$$\left(\frac{\lambda_i a_i}{\lambda_i m_i}\right)^{\sigma_i} + \left(\frac{\lambda_i b_i}{\lambda_i n_i}\right)^{\sigma_i} = 1.$$

For the simplicity of notation, we can rewrite $\lambda_i a_i$ and $\lambda_i b_i$ as new a_i and b_i, and these new a_i and b_i will satisfy

$$\left(\frac{a_i}{\lambda_i m_i}\right)^{\sigma_i} + \left(\frac{b_i}{\lambda_i n_i}\right)^{\sigma_i} = 1,$$

and the technology is still

$$Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i}.$$

After such a transformation, we see that the effect of such kind of technology choice cost makes the technology frontier shrink. That is, after considering the technology choice cost, the m_i and n_i all shrink.

Obviously, under different technology frontiers, original or shrunk, the performance of the economy is different. In general, under the shrunk technology frontier, the technical upgrading in each industry is slower than that under the original technology frontier. That is, the cost of technology choice impede the technical upgrading and hence the structural change of the economy.

This will induce the mismatch between industrial structure and factor endowment, in short-term, or even in medium-term. Such a phenomenon could be observed in reality. To eliminate or at least abate this mismatch, well-developed macroeconomic environment (well-defined property rights and market order) and good policies will be helpful.

4. Calibration and simulation

In this section, we use the NBER-CES data set to compute the technology choice and technology frontier of each industry from observed data on endowments and factor returns. Further, we show empirical patterns that are consistent with our model predictions. Using the estimated parameters, we simulate the model quantitatively and conduct counterfactual analysis.

4.1. Data description

The key variables we use to compute technology choices include value added y, real capital stock k, total employment l, as well as the wage payment in each industry each year. We divide total wage payments by employment for each industry to obtain the wage. The capital rental price is computed as $(y - wl)/k$. The value added is deflated by the producer price index. The capital stock is provided in real terms. Table 3 summarizes the data we use, and the total number of observations is 25,386 over 54 years.

Table 3
Descriptive summary of the data.

Variable	Obs	Mean	Std. dev.	Min	Max
Employment (1,000)	25,386	34.46	44.52	0.2	559.9
Wage payment (US$, 1,000s)	25,386	749.76	1,270.19	2.9	16,162.9
Capital stock (US$, 1,000s)	25,386	2,793.32	6,475.77	4.1	133,347.3
Value added (US$, 1,000s)	25,386	2,275.96	4,941.65	9.7	111,665.7

Table 4
Coefficients.

Parameter	σ	τ	ρ_0	ε
Value	−0.30	−0.50	−1.00	0.43

4.2. Computation of technology choice

In this subsection, we compute the technology choice of a_i, b_i. From Section 3.4, we know that for any $i \in (0,1)$,

$$a_i = \frac{r}{p_i}\left(\frac{rK_i}{rK_i + wL_i}\right)^{\frac{1-\rho_i}{\rho_i}}, \quad b_i = \frac{w}{p_i}\left(\frac{wL_i}{rK_i + wL_i}\right)^{\frac{1-\rho_i}{\rho_i}}.$$

Hence, we solve for a_i and b_i using data on K_i, L_i, r, and w, after calibrating parameter ρ_i.

We assume that $\rho_i \equiv \rho_0$ for all $i \in (0,1)$, where $\rho_0 \in (0,1)$ is some constant. The parameter ρ_0 is determined by the elasticity of substitution between effective capital and labor, $1/(1-\rho_0)$, which has attracted a considerable amount of attention in the macro literature. Leonledesma et al. (2010) and Knoblach and Stockl (2020) provide summaries of capital–labor substitution elasticity in production for the United States. Most contributions to the literature suggest that the value is greater than 0.5 and less than 1, suggesting that capital and labor are complements in the production function, instead of substitutes. For example, Bodkin and Klein (1967) suggest 0.5 to 0.7, Klump et al. (2007) suggest 0.56, Leonledesma and Satchi (2019) suggest 0.2, and so forth. Wang et al. (2018) and Knoblach et al. (2020) provide nice reviews. Knoblach et al. (2020) utilize 738 estimates from 41 studies published between 1961 and 2016 and find the estimates of long-run elasticity lie in the range between 0.6 and 0.7. In our excise, we show that the results are robust using different values of $1/(1-\rho_0)$ between 0.4 and 0.7.

4.3. Backing out the technology frontiers

After computing technology choices a_i and b_i, we follow Caselli and Coleman (2006) to estimate each industry's technology frontiers $(a_i/m_i)^{\sigma_i} + (b_i/n_i)^{\sigma_i} = 1$ in each year.

In this section, we assume $\sigma_i = \sigma$ for all i, where σ is some constant. From Section 3.3, we know that for any i, $\eta_i^{\sigma-\rho_0} = k_i^{\rho_0}\gamma_i^{-\sigma}$, it follows that $\log \eta_i = \frac{\rho_0}{\sigma-\rho_0}\log k_i - \frac{\sigma}{\sigma-\rho_0}\log \gamma_i$. Then, we conduct the following regression: $\log \eta_{it} = \beta \log k_{it} + \epsilon_{it}$, where β is the regression coefficient and ϵ_{it} is the error.

From this regression, we obtain estimates of σ and γ_i. Together with the formula for the technology frontier, we have estimates of m_i and n_i for each industry in each year. Table 4 summarizes our estimates of the parameters. The values are consistent with the technical assumptions in Section 3.

4.4. Change in technology choice with increasing capital endowment

We test Proposition 2 of the model. Table 5 shows the results of a regression where the dependent variable is technology choice η_{it} (in log), and the independent variables are aggregate endowment $\overline{K}/\overline{L}$ (in log) and the technology frontiers m_{it}/n_{it} (in log) we computed in Section 4.3. We control for industry fixed effects in each regression, so the coefficients capture the within-industry response of technology

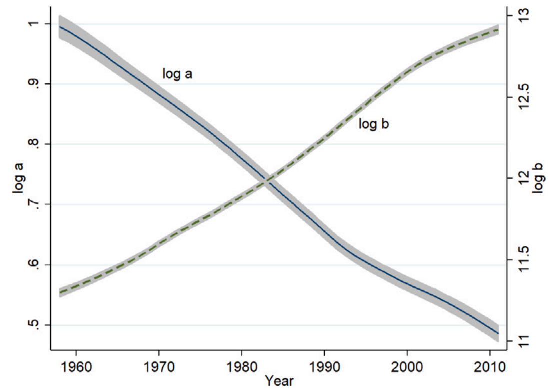

Fig. 6. Technology choices in the United States.

Table 5
Technology Choices, Endowments, and Frontiers.

	(1) $log(\eta)$	(2) $log(\eta)$	(3) $log(\eta)$	(4) $log(\eta)$
$1/(1-\rho_0)$	0.4	0.5	0.6	0.7
$log(\overline{K}/\overline{L})$	−0.485***	−0.554***	−0.656***	−0.826***
	(−17.76)	(−19.77)	(−22.49)	(−26.23)
$log(m/n)$	−0.0731***	−0.0492***	−0.0212*	0.0122
	(−6.24)	(−4.07)	(−1.68)	−0.92
N	25,386	25,386	25,386	25,386
r2	0.313	0.317	0.349	0.424

choices to the increase in aggregate capital endowment and changes in technology frontiers.

Columns (1) to (4) use different calibrations of coefficient ρ_0, and we find that the results are robust to the different values. The coefficients on capital endowment and technology frontier m_{it}/n_{it} are negative, which is consistent with the model. With more abundant capital endowment, U.S. industries choose more labor-efficient technologies.

Fig. 6 is a more straightforward illustration of the result. The graph shows the local polynomial fitness curves of the technology choices a and b for all industries over time. Consistent with our regression findings, we observe increasing labor-augmenting factor b and decreasing capital-augmenting factor a.

4.5. Simulation

In this section, we simulate our model and conduct a counterfactual test of how the structural change would be affected by firms' technology choices. The simulation is based on calibration of exogenous parameters. First, we use the parameters we estimated in the last section. Then, we further calibrate those parameters to include demand shifter θ_{it} for each industry i in year t. We calibrate the elasticity of substitution between differentiated goods, $1/(1-\rho_0)$, as 0.76, following Acemoglu and Guerrieri (2008).

We group all the industries into five categories, from labor intensive to capital intensive, based on the average capital intensity in the whole sample period. We plot the subsequent capital intensities of the five regrouped industries in Fig. 7. The figure shows that capital intensity is increasing in each group of industries, and the relative order remains stable overtime.

We estimate the technology frontier using the same method as in Section 4.3, Fig. 8 shows the pattern of the technology frontier parameter $\gamma_{it} = m_{it}/n_{it}$. A rising γ indicates that the capital-augmenting technology frontier grows faster than the labor-augmenting technology frontier. We can see that γ was generally declining in each industry, except that industry 1 (the most labor-intensive industry) experienced an increasing γ roughly after 2000. The results suggest that although a

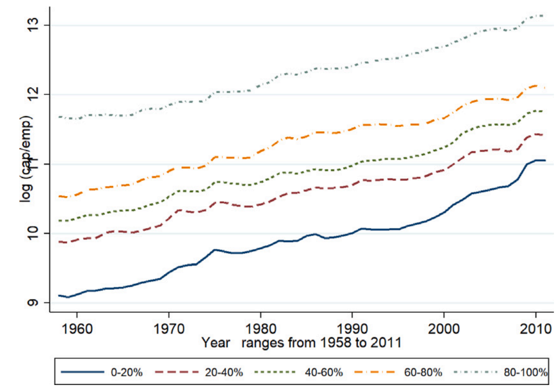

Fig. 7. Capital intensities in the United States.
Note: All the industries are grouped into five categories, from labor-intensive to capital-intensive. Each line represents one regrouped industry, where 0%–20% represents the most labor-intensive industry, and 80%–100% represents the most capital-intensive industry.

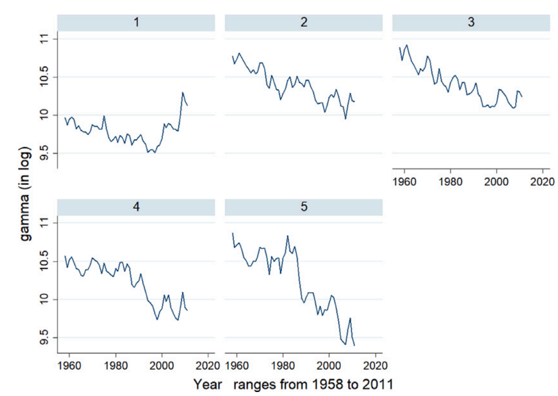

Fig. 8. Estimated technology frontiers m_{it}/n_{it} in the United States.

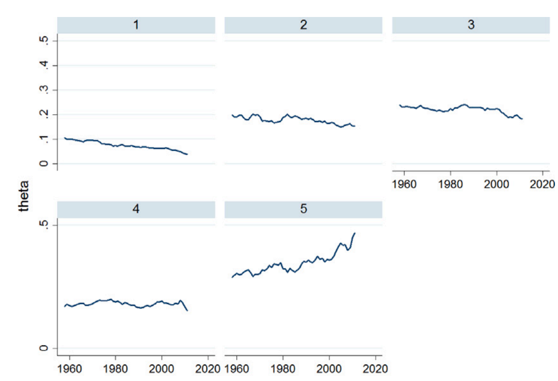

Fig. 9. Calibrated demand shifter θ_{it} in the United States.

declining γ_{it} tends to reduce capital intensity as well as increase a_{it}/b_{it} when capital and labor are complementary, the increasing aggregate capital stock (per capita) dominates and, thus, induces increasing capital intensity, as well as declining a_{it}/b_{it}, in U.S. industries during 1958 to 2011.

We carry out the following steps to simulate the model. The first step is to use the data on industrial value-added share to back out the

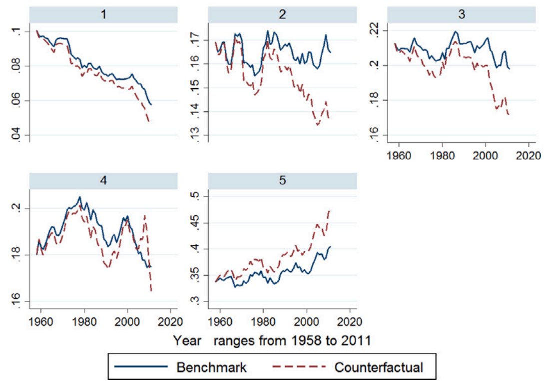

Fig. 10. Counterfactual structural change (Value-Added Shares).

demand shifter θ_i for each industry, using Eq. (3). The calibrated demand shifters θ_{it} are shown in Fig. 9. We find that the demand shifters are relatively more stable than the technology frontiers, showing a declining pattern in industries 1, 2, and 3; a stable pattern in industry 4; and an increasing pattern in industry 5.

The second step is to solve the factor price. In each counterfactual analysis, the factor price is solved by Eq. (2). In the model, we assume a uniform factor price in all industries, while in the data, each industry has heterogeneous factor prices.[13] The third step is to solve for the counterfactual variables of capital, labor, and value-added share, using the equations Section 3.3.

We take m and n in the first year as fixed for the whole sample period and solve the model. Fig. 10 shows the value-added share of each industry over time, where the solid line is the benchmark model and the dashed line is the counterfactual model. The figure shows that structural change would be faster if the technology frontiers were kept at their initial levels, such that the market shares of labor intensive sectors start to decline earlier, and that of the capital intensive sectors start to grow earlier.

5. Conclusion

This paper characterizes the evolution of the industrial sector during economic growth both empirically and theoretically. The key findings are that, first, technology in each manufacturing industry, as well as the whole industrial sector, becomes more capital-intensive as the economy becomes more capital abundant. Second, the industrial structure is unimodal, with a leading industry at any time in the process of economic development; with the accumulation of capital endowment, the industrial structure shifts more and more to capital-intensive industries, and the leading industry is taken over by a new leading industry continuously.

Our model shows that the factor endowment structure is crucial in determining technology choice and industrial structure. The dynamic model captures the interaction among endowment structure, technological choices, and industrial structure. The technology choices and industrial structure are determined by given endowments in each period. As time goes by, the economy accumulates capital, and the aggregate endowment structure becomes more capital abundant. Notably, the optimal technology choices and industrial structure guarantee the fastest speed in accumulating capital. In this way, our model captures the dynamic process of industrialization.

[13] In the counterfactual analysis, we use the model predictions under the baseline parameters for comparison with the model predictions under the counterfactual parameters.

This paper has policy implications concerning the link between comparative-advantage-defying strategies and welfare loss. Our model shows that in the best-first economy, the structural change of an economy follows its comparative advantage, with the capital intensity of the leading industry determined by the total endowments. However, under the catching-up strategies, which aim to advance capital-intensive industries when the total capital endowment is scarce, the economy deviates from the first-best allocation, and the capital accumulation speed becomes slower than optimal. Therefore, the comparative-advantage-defying strategies induce welfare losses.

The second policy implication concerns the link between the cost of technological choices and the role of government intervention. The benchmark model in our paper abstracts away the costs of technological choices. To account for the costs in the real world, we provide an extension of the model where technology changes incur costs. As we show, a small cost of technological choices delays industrial upgrading. However, industrialization could come to a complete stop when the cost is high. The costs could take the forms of information friction, the lack of infrastructure, and insufficient human capital in facilitating structural change from small-scale, labor-intensive industries to large-scale, capital-intensive industries. Then there will be a need for government interventions to provide information, improve hard and soft infrastructures, and improve human capital, among others, in industrial upgrading.

The model we build has elegant closed solutions, providing a workhorse for multi-industry structural analysis. By incorporating various frictions and market failures into the model in the future, we will be able to discuss different policy options and the role of the state in the economy. Further, the model can be used to explore issues related to international trade and technology spillovers across countries at different levels of development.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A. Corner solution

In the benchmark setting, an interior solution occurs, in which the assumption $\sigma_i > \tau_i$ is essential.

Here, for simplicity and comparison, we consider the discrete case, where the economy has only n intermediate sectors, and $\rho_i > 0$, $\sigma_i > 0$ for all $i = 1, \ldots, n$. Further, replacing assumption (3), we make another assumption: for any $i = 1, \ldots, n$,

$$\varepsilon_i > \rho, \quad \sigma_i < \tau_i.$$

In this case, by a method similar to the one used in the proof of Theorem 1 in Appendix C, we see that each firm in the intermediate sectors has a corner solution in its technology choice problem, that is, for a firm in industry i, the optimal choice of a_i, b_i is $a_i = m_i, \quad b_i = 0$, and accordingly, $K_i > 0, \quad L_i = 0$; or $a_i = 0, \quad b_i = n_i$, and accordingly, $K_i = 0, \quad L_i > 0$; or both.

In the last case, both corners are optimal, in this industry, different firms may choose different technologies. In this case, the assumption "in each industry, there is only one firm, and it can take only one technology" is not suitable, because under this assumption, the general equilibrium may not exist. Therefore, replacing it, we make another assumption that in each industry, there are two firms, which take different corner technologies.

And, because the production functions are all first-order homogeneous, an equivalent assumption is as follows: there is only one firm in each industry in the intermediate sectors, but it can take the two corner technologies simultaneously to produce its products.

Under this assumption, we have that for the firm in the industry $i \in \{1, \dots, n\}$, if the capital and labor it demands are K_i, L_i respectively, then, the output it produces is $Y_i = m_i K_i + n_i L_i$, and if $K_i > 0$, then, it must take the technology $a_i = m_i, \quad b_i = 0$; if $L_i > 0$, then, it must take the technology $a_i = 0, \quad b_i = n_i$. The concept of general equilibrium can be defined similarly. Hence, to express the equilibrium, without any loss, we omit writing out a_i, b_i and C_i explicitly.

For simplicity, we make a further assumption that $\rho = 0$, and hence, the production function of the final good sector is of Cobb–Douglas form, that is,

$$Y = \prod_{i=1}^{n} Y_i^{\theta_i}.$$

We say that industry j is more capital intensive potentially (or higher) than industry i, if $\gamma_j > \gamma_i$.

Now, we rank $\{\gamma_1, \dots, \gamma_n\}$ in increasing order. For simplicity, we assume that

$$\gamma_1 < \gamma_2 < \cdots < \gamma_n.$$

That is, from industry 1 to industry n, the potential capital intensities are increasing.

For any $j = 1, \dots, n-1$, denote

$$H_j = \prod_{i \le j} (\theta_i n_i)^{\theta_i} \prod_{i > j} (\theta_i m_i)^{\theta_i}, \quad q_j = \sum_{i \le j} \theta_i,$$

$$B_j = \frac{1 - q_j}{q_j}, A_j = \frac{B_j}{\gamma_j}, \quad A'_j = \frac{B_j}{\gamma_{j+1}},$$

and set $A'_0 =: \infty, A_n =: 0$. We have that $\infty = A'_0 > A_1 > A'_1 > \cdots > A_{n-1} > A'_{n-1} > A_n = 0$. Now, we state our result.

Theorem 1'. *The equilibrium* $\left(K_i, L_i; p_i, r, \omega\right)_{i=1,\dots,n}$ *is unique, which is determined as follows.*

(i) If for some $j \in \{1, \dots, n-1\}$, $A_j \ge K \ge A'_j$, *then,*

$$K_i = 0, \quad L_i = \frac{\theta_i}{q_j}, \quad \forall i \le j,$$

$$L_i = 0, \quad K_i = \frac{\theta_i}{1 - q_j} K, \quad \forall i > j.$$

(ii) If for some $j \in \{1, \dots, n\}$, $A'_{j-1} > K > A_j$, *then,*

$$K_i = 0, \quad L_i = \theta_i \left(\gamma_j K + 1\right), \quad \forall i < j,$$

$$L_i = 0, \quad K_i = \theta_i \left(K + 1/\gamma_j\right), \quad \forall i > j,$$

$$K_j = q_j K - \frac{1 - q_j}{\gamma_j}, \quad L_j = (1 - q_{j-1}) - \gamma_j q_{j-1} K.$$

(iii)

$$r = \sum_j \left(\frac{B_j}{K}\right)^{q_j} H_j I \left(A'_j \le K \le A_j\right) + \sum_j \gamma_{j+1}^{q_j} H_j I \left(A_{j+1} < K < A'_j\right);$$

$$\omega = \sum_j \left(\frac{K}{B_j}\right)^{1-q_j} H_j I \left(A'_j \le K \le A_j\right) + \sum_j \gamma_{j+1}^{q_j - 1} H_j I \left(A_{j+1} < K < A'_j\right);$$

$$p_i = \begin{cases} \omega / n_i, & \text{if} \quad K_i > 0, \\ r / m_i, & \text{if} \quad L_i > 0. \end{cases}$$

(iv) (K, C) *is the unique solution of*

$$\max_{(K,C)} \int_0^\infty e^{-\rho t} C^{1-\theta} dt,$$

$$\text{s.t.} \quad \dot{K} = W(K) - C,$$

$$K(0) = K_0,$$

where

$$W(K) =: \sum_j K^{1-q_j} \frac{B_j^{q_j}}{1 - q_j} H_j I \left(A'_j \le K \le A_j\right) + \sum_j \gamma_{j+1}^{q_j} \left(K + \frac{1}{\gamma_{j+1}}\right) H_j I \left(A_{j+1} < K < A'_j\right).$$

Clearly, W *is smooth, concave, and strictly increasing.*

Summing up the above analysis, in this setting, we have the following proposition.

Proposition 1' (*Pattern of Technology Upgrading*). *Along with the increase in the capital per capita of the society, all the industries will upgrade their technology sequentially, and each industry (except the first and last industries) will experience three phases of technology upgrading: pure labor technology, mixed technology, and pure capital technology. The first industry experiences only two phases: pure labor technology, and mixed technology; the last industry experiences only two phases: mixed technology, and pure capital technology. At any level of capital per capita, there is only one industry taking the mixed technology, all lower industries take pure labor technology, and all higher industries take pure capital technology. Along with the increase in capital per capita from 0 to ∞, the order of technical upgrading is from higher industries to lower industries one by one sequentially.*

Appendix B. Leontief case

In the basic setting in Section 3.1, holding all the assumptions, and setting $\rho_i = -\infty$ for some (may or may not be all) $i \in [0, 1]$, the technologies in these industries will be reduced to Leontief type. That is, for these industries, the production functions will be

$$Y_i = \min\{a_i K_i, b_i L_i\}.$$

In this case, for these industries, we have

$$\tau_i = -1, \quad \varepsilon_i = -\sigma_i, \quad \delta_i = -\frac{\sigma_i}{1 + \sigma_i},$$

and, assumption (3) will be reduced to

$$\rho < -\sigma_i < 1.$$

In this case, all of our main results will remain true. That is, the Leontief case is a special case in our general setting. More precisely, even if the production functions in some of the industries are changed from the ordinary CES function to a Leontief function, the equilibrium will also exist and is unique, and it can be determined by the same method as in Theorem 1.

In the end, we discuss the Leontief case in the problem of technology choice in a static setting. We first consider a one-sector model in which the production function is of Leontief type:

$$Y = \min\{aK, bL\},$$

where Y is the consumption good; K, L are capital and labor, respectively, $a > 0, b > 0$ are technology parameters, and the set of all possible (a, b) is denoted as T, which is the technology set. Suppose there is only one individual, owning the initial capital endowment $\overline{K}$ and labor endowment $\overline{L}$, and denote $\overline{k} = \overline{K}/\overline{L}$.

If T is only a single-point set, then, obviously, the Walrasian equilibrium exists always. We denote the prices of capital and labor as r and ω, respectively, and normalize the price of the consumption good as 1.

If $b/a = \overline{k}$, then the equilibria are multiple, and (r, ω) is the equilibrium prices, if and only if it satisfies

$$1 = \frac{r}{a} + \frac{\omega}{b}.$$

If $b/a > \bar{k}$, then the unique Walrasian equilibrium prices are $r = a$, $\omega = 0$. In this case, there exists free disposal of labor. That is, with respect to this technology, the social labor supply is excessive. The equilibrium labor demand is any $L \in [\bar{K}a/b, \bar{L}]$.

If $b/a < \bar{k}$, then the unique Walrasian equilibrium prices are $r = 0$, $\omega = b$. In this case, there exists free disposal of capital. That is, with respect to this technology, the social capital supply is excessive. The equilibrium capital demand is any $K \in [\bar{L}b/a, \bar{K}]$.

If the technology set T is a two-point set, say, $T = \{(a_1, b_1), (a_2, b_2)\}$, and suppose that

$$\frac{b_1}{a_1} < \frac{b_2}{a_2},$$

then, if only one technology can be taken, then, the Walrasian equilibrium without free disposal exists, if and only if there is an $i \in \{1, 2\}$ such that

$$\frac{b_i}{a_i} = \bar{k}.$$

If the two technologies can be chosen simultaneously, then, the Walrasian equilibrium without free disposal exists, if and only if

$$\frac{b_1}{a_1} \le \bar{k} \le \frac{b_2}{a_2}.$$

Otherwise, there only exists Walrasian free disposal equilibrium.

This discussion can be extended to multiple-sector case, and similar results are obtained. As an example, here we discuss a two-sector model, and we only discuss the pure technology case, that is, any firm is only allowed to take one technology. For simplicity, we assume that in any one of the two sectors, there is only one firm, and in this economy, there is only one individual with endowments $\bar{K}, \bar{L}$, and let $\bar{k} = \bar{K}/\bar{L}$.

Suppose the individual's utility function is

$$U(C_1, C_2) = C_1^{\theta_1} C_2^{\theta_2},$$

where $\theta_1 > 0$, $\theta_2 > 0$ and $\theta_1 + \theta_2 = 1$. And suppose that for any $i \in \{1, 2\}$, in industry i, the possible production function is

$$Y_i = \min\{a_i K_i, b_i L_i\},$$

and the technology set is T_i.

Then, it can be proved that the Walrasian equilibrium without free disposal exists, if and only if there exist $(a_1, b_1) \in T_1$, $(a_2, b_2) \in T_2$ such that $\bar{k}$ is located between

$$\theta_1 \frac{b_1}{a_1} + \theta_2 \frac{b_2}{a_2} \qquad \text{and} \qquad \left(\theta_1 \frac{a_1}{b_1} + \theta_2 \frac{a_2}{b_2}\right)^{-1}.$$

Otherwise, there exist only Walrasian free disposal equilibria.

After all, in the Leontief case, if free disposal is allowed, then the equilibrium exists always. If free disposal is not allowed, then the equilibrium may or may not exist, which depends, partly, on the structure of the technology set: if the technology set is too poor, then the equilibrium is not likely to exist; if it is quite rich, then the equilibrium is likely to exist.

This is a difference between the ordinary CES case in Section 3.1 and the Leontief case here. In the setting in Section 3.1, the equilibrium without free disposal and the equilibrium with free disposal are equivalent.

Appendix C. Proofs

Proof of Theorem 1. One can verify that the given solution is really an equilibrium, from which the sufficiency follows. In the sequel, we prove the necessity. Suppose that

$$\left(r(t), \omega(t), p_i(t), a_i(t), b_i(t), K_i(t), L_i(t), Y_i(t), Y(t), K(t), C(t)\right)_{i\in(0,1),t\ge 0}$$

is an equilibrium.

For neatness of expression, we omit writing out (t), for example, we write r, instead of $r(t)$.

In the sequel, fix $t \ge 0$ arbitrarily. For any $i \in (0, 1)$, denote

$$r_i = \frac{r}{p_i}, \quad \omega_i = \frac{\omega}{p_i}, \quad k_i = \frac{K_i}{L_i}, \quad \eta_i = \frac{a_i}{b_i}, \quad x_i = \left(\eta_i k_i\right)^{\rho_i}, \quad s_i = \left(\gamma_i z\right)^{\delta_i},$$

where

$$z =: \frac{\omega}{r}.$$

By solving the profit maximization problem for the firm in the final good sector, we get that for any $i \in (0, 1)$,

$$p_i Y_i^{1-\rho} = \theta_i Y^{1-\rho}, \tag{10}$$

therefore,

$$\int_0^1 \left(\theta_i p_i^{-\rho}\right)^{1/(1-\rho)} di = 1. \tag{11}$$

For any $i \in (0, 1)$, we know

$$0 = \max_{(a_i,b_i,K_i,L_i,Y_i)} \left\{Y_i - r_i K_i - \omega_i L_i\right\}, \tag{12}$$

s.t.

$$Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i}, \qquad \left(\frac{a_i}{m_i}\right)^{\sigma_i} + \left(\frac{b_i}{n_i}\right)^{\sigma_i} = 1.$$

We first solve the embedded cost minimization problem:

$$Z_i =: \min_{(K_i,L_i)} \left\{r_i K_i + \omega_i L_i\right\},$$

s.t.

$$Y_i = \left((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}\right)^{1/\rho_i},$$

the solution of which satisfies

$$r_i K_i = \mu_i \frac{(a_i K_i)^{\rho_i}}{(a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}} Y_i, \tag{13}$$

$$\omega_i L_i = \mu_i \frac{(b_i L_i)^{\rho_i}}{(a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}} Y_i, \tag{14}$$

for some Lagrange multiplier $\mu_i > 0$, and hence,

$$Z_i = \mu_i Y_i.$$

From (13), (14), we get

$$\frac{r_i K_i}{\omega_i L_i} = \left(\frac{a_i K_i}{b_i L_i}\right)^{\rho_i} = \left(\frac{a_i/r_i}{b_i/\omega_i}\right)^{\tau_i}, \tag{15}$$

which, together with (14) again, gives

$$\begin{aligned}\omega_i L_i &= \mu_i b_i L_i \left(1 + \left(\frac{a_i K_i}{b_i L_i}\right)^{\rho_i}\right)^{1/\rho_i - 1} \\ &= \mu_i b_i L_i \left(1 + \left(\frac{a_i/r_i}{b_i/\omega_i}\right)^{\tau_i}\right)^{1/\tau_i},\end{aligned}$$

which yields

$$\mu_i = \left(\left(\frac{a_i}{r_i}\right)^{\tau_i} + \left(\frac{b_i}{\omega_i}\right)^{\tau_i}\right)^{-1/\tau_i}.$$

Then, (12) becomes

$$0 = \max_{(a_i,b_i,Y_i)} \left(1 - \left(\left(\frac{a_i}{r_i}\right)^{\tau_i} + \left(\frac{b_i}{\omega_i}\right)^{\tau_i}\right)^{-1/\tau_i}\right) Y_i,$$

s.t.

$$\left(\frac{a_i}{m_i}\right)^{\sigma_i} + \left(\frac{b_i}{n_i}\right)^{\sigma_i} = 1, \quad Y_i \ge 0,$$

then,

$$1 = \max_{(a_i,b_i)} \left(\left(\frac{a_i}{r_i}\right)^{\tau_i} + \left(\frac{b_i}{\omega_i}\right)^{\tau_i}\right)^{1/\tau_i},$$

s.t.

$$\left(\frac{a_i}{m_i}\right)^{\sigma_i} + \left(\frac{b_i}{n_i}\right)^{\sigma_i} = 1,$$

the solution of which satisfies

$$\left(\frac{a_i}{r_i}\right)^{\tau_i} = \left(\frac{a_i}{m_i}\right)^{\sigma_i} = \left(\frac{m_i}{r_i}\right)^{\delta_i} = \frac{s_i}{1+s_i}, \tag{16}$$

$$\left(\frac{b_i}{\omega_i}\right)^{\tau_i} = \left(\frac{b_i}{n_i}\right)^{\sigma_i} = \left(\frac{n_i}{\omega_i}\right)^{\delta_i} = \frac{1}{1+s_i}. \tag{17}$$

And hence, $\mu_i = 1$. Therefore,

$$Y_i = r_i K_i (1 + x_i^{-1}) = \omega_i L_i (1 + x_i). \tag{18}$$

It follows that for any $i \in (0,1)$,

$$s_i = \left(\frac{\eta_i}{\gamma_i}\right)^{\sigma_i} = \left(\eta_i z\right)^{\tau_i}.$$

From (15), we have

$$k_i = z x_i.$$

Thus,

$$x_i = \left(\eta_i k_i\right)^{\rho_i} = \left(\eta_i z x_i\right)^{\rho_i} = \left(\eta_i z\right)^{\tau_i} = s_i.$$

From (16), (17), we have

$$\begin{aligned} a_i &= m_i \left(1 + s_i^{-1}\right)^{-1/\sigma_i}, \\ b_i &= n_i \left(1 + s_i\right)^{-1/\sigma_i}, \\ r_i &= m_i \left(1 + s_i^{-1}\right)^{1/\delta_i}, \\ \omega_i &= n_i \left(1 + s_i\right)^{1/\delta_i}. \end{aligned} \tag{19}$$

From (10), (18), (19), we obtain

$$K_i^{1-\rho} = \theta_i m_i^{\rho} (1 + x_i^{-1})^{\rho/\varepsilon_i - 1} Y^{1-\rho} / r.$$

Since

$$\int_0^1 K_i di = K,$$

then,

$$Y r^{1/(\rho-1)} = \frac{K}{\int_0^1 \left(\theta_i m_i^{\rho} (1 + x_i^{-1})^{\rho/\varepsilon_i - 1}\right)^{1/(1-\rho)} di}.$$

Therefore,

$$K_i = \frac{\left(\theta_i m_i^{\rho} (1 + x_i^{-1})^{\rho/\varepsilon_i - 1}\right)^{1/(1-\rho)}}{\int_0^1 \left(\theta_j m_j^{\rho} (1 + x_j^{-1})^{\rho/\varepsilon_j - 1}\right)^{1/(1-\rho)} di} K, \tag{20}$$

Analogously,

$$L_i = \frac{\left(\theta_i n_i^{\rho} (1 + x_i)^{\rho/\varepsilon_i - 1}\right)^{1/(1-\rho)}}{\int_0^1 \left(\theta_j n_j^{\rho} (1 + x_j)^{\rho/\varepsilon_j - 1}\right)^{1/(1-\rho)} dj}, \tag{21}$$

Dividing (20) by (21) on both sides simultaneously, and noticing $k_i = z x_i$ and $x_i = (\gamma_i z)^{\delta_i}$, we get $K = \varphi(z)$, where φ is defined in Theorem 1.

From (11), (19), we get

$$r = \left(\int_0^1 \theta_i^{1/(1-\rho)} \left[(z^{-1} n_i)^{\delta_i} + m_i^{\delta_i}\right]^{\rho/[\delta_i(1-\rho)]} di\right)^{(1-\rho)/\rho} = \psi(z),$$

$$\omega = rz, \quad p_i = r\left((z^{-1} n_i)^{\delta_i} + m_i^{\delta_i}\right)^{-1/\delta_i}.$$

So far, we see that the variables (r,ω), $(p_i, a_i, b_i, K_i, L_i)_{i\in(0,1)}$ are all determined uniquely by K, and $(Y_i)_{i\in(0,1)}$ and Y are also determined uniquely by K through

$$Y_i = \left((b_i K_i)^{\rho_i} + (a_i L_i)^{\rho_i}\right)^{1/\rho_i}, \quad Y = \left(\int_0^1 \theta_i Y_i^{\rho} di\right)^{1/\rho}.$$

Clearly, for any $i \in (0,1)$,

$$p_i = \theta_i \left(\frac{Y_i}{Y}\right)^{\rho-1}, \quad \frac{\partial Y_i}{\partial K_i} = \frac{r}{p_i}, \quad \frac{\partial Y_i}{\partial L_i} = \frac{\omega}{p_i}.$$

In addition, from (19), we get that for any $i \in (0,1)$,

$$s_i \frac{d a_i}{d s_i} + \eta_i \frac{d b_i}{d s_i} = 0,$$

which implies

$$\frac{\partial Y_i}{\partial a_i}\frac{d a_i}{dK} + \frac{\partial Y_i}{\partial b_i}\frac{d b_i}{dK} = 0.$$

Noticing

$$\int_0^1 K_i di = K, \quad \int_0^1 L_i di = 1,$$

we get

$$\int_0^1 \frac{dK_i}{dK} di = 1, \quad \int_0^1 \frac{dL_i}{dK} di = 0.$$

Therefore,

$$\begin{aligned} \frac{dY}{dK} &= \int_0^1 \theta_i \left(\frac{Y_i}{Y}\right)^{\rho-1} \left[\frac{\partial Y_i}{\partial K_i}\frac{dK_i}{dK} + \frac{\partial Y_i}{\partial L_i}\frac{dL_i}{dK}\right] di \\ &= \int_0^1 p_i \left[\frac{r}{p_i}\frac{dK_i}{dK} + \frac{\omega}{p_i}\frac{dL_i}{dK}\right] di \\ &= r\int_0^1 \frac{dK_i}{dK} di + \omega \int_0^1 \frac{dL_i}{dK} di \\ &= r. \end{aligned}$$

We know that $Y = W(K)$, $r = f(K)$, and hence, $W'(K) = f(K)$.

Finally, by the definition of equilibrium, (K,C) is a solution of problem:

$$\begin{aligned} &\max_{(K,C)} \int_0^\infty e^{-\varrho t} C^{1-\theta} dt, \\ &\text{s.t.} \quad \dot{K} = rK + \omega - C, \\ &\qquad K(0) = K_0. \end{aligned}$$

Obviously, along the equilibrium path,

$$rK + \omega = W(K),$$

then, (K,C) is a feasible path for the problem $\mathbb{P}'$:

$$\begin{aligned} &\max_{(K,C)} \int_0^\infty e^{-\varrho t} C^{1-\theta} dt, \\ &\text{s.t.} \quad \dot{K} = W(K) - C, \\ &\qquad K(0) = K_0. \end{aligned}$$

Furthermore, noting that for any $K > 0$ and any $Z > 0$,

$$W(K) \le f(Z)K + g(Z),$$

one can easily prove that (K,C) is really the solution of problem $\mathbb{P}'$ by the method of contradiction. Theorem 1 is proved.

The proof of Theorem 1' is similar to that of Theorem 1, hence, omitted.

Appendix D. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.strueco.2023.03.002.

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