The shadow price of nitrogen

A dynamic analysis of nitrogen-induced soil acidification in China

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Abstract

Purpose – The most recent and prestigious scientific research shows that nitrogen leaching caused by over-used nitrogen fertilizer rapidly acidifies all soil types in China, revolutionizing the basic understanding of the mechanism of soil acidification. The purpose of this paper is to study the impact of nitrogen on soil acidity over the long run, which is the shadow price of nitrogen.

Design/methodology/approach – In a discrete dynamic programming model, this paper compares the nitrogen application and soil pH between optimal nitrogen control that takes the shadow price of nitrogen into consideration and myopic nitrogen control that ignores that shadow price. Using a five-year panel experimental data on a rapeseed-rice rotation, this paper simulates and numerically solves the dynamic model.

Findings – Both theoretically and empirically, this paper shows that the over-use of nitrogen and the decline in soil pH are explained by ignorance of the shadow price of nitrogen. Compared with optimal nitrogen control, myopic nitrogen control applies more nitrogen in total, resulting in lower soil pH. In addition, over-use in the first season contributes to soil acidification and the carry-over effects mitigate that problem.

Originality/value – This paper enriches the literature by extending the study of the environmental impact of nitrogen leaching to its impact on the long-term loss in agricultural production, providing a new theoretical framework in which to study soil acidification rather than conventionally treating soil acidification as a secondary consequence of acid rain, and showing the possibility of using nitrogen control to mitigate soil acidification when lime applications are not feasible due to socio-economic constraints.

Keywords Dynamic programming, Nitrogen control, Soil acidification, Soil conservation

Paper type Research paper

Introduction

The average nitrogen application in China for each production season is around 22 kg/mu, which is about three times the world's average application rate and around 2.5 times the average application rate in the USA and Europe (MOA, 2015). Between 1980 and 2007, nitrogen application in China increased by 191 percent (Guo *et al.*, 2010), making China the biggest user of nitrogen fertilizer in the world (Norse and Zhang, 2010). The massive use of nitrogen is encouraged by a series of national policies on fertilization. From the supply side, the policies that are favorable to nitrogen production include but are not limited to a reduction in transportation costs through the railway system, heavy subsidies of the use of electricity and gasoline in nitrogen production and zero-interest-rate loans for winter nitrogen restocking (MOA, 2015). From the demand side, a price ceiling is set at 400 RMB/ton along with a subsidy of 100 RMB/ton that is delivered directly to nitrogen buyers (MOA, 2015).

The massive use of nitrogen improved agricultural productivity in China between 1980 and 2007. However, China's grain production declined after 2007 along with a continuously increasing nitrogen application rate (Zhang *et al.*, 2008). Meanwhile, after 30 years of monitoring fixed plots all over China, the most recent scientific research, Guo *et al.* (2010), published in *Science*, shows that nitrogen leaching caused by over-applied nitrogen fertilizer

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acidifies topsoil pH by 0.14 to 0.80 between 1980 and 2007 in all five soil types in China. While soil quality is always vaguely defined (Brady and Weil, 2002), soil pH is the most important and thorough index of soil quality that directly reveals what crops can be grown and what cannot (Brady and Weil, 2002). Soil acidification could devastate agricultural production. A 0.5 decrease in soil pH increases the amount of dissolved toxic elements 1,000 times (Bolan and Hedley, 2003; Brady and Weil, 2002). Thus, a small decline in soil pH immediately leads to higher levels of toxic elements in the topsoil (Zhang *et al.*, 2008), which burns roots and results in stunted, discolored growth and poor yield (Meng *et al.*, 2004). In the 2010s, nitrogen-induced soil acidification has greatly threatened the sustainability of agricultural productivity in China in all top 13 commodity grain production regions, accounting for 80 percent of national grain production (MOA, 2004).

Inasmuch as over-used nitrogen induces soil acidification, China's fertilization problem changed from a shortage in supply and low purchasing capacity (Ye, 1993) to excessive demand (Guo *et al.*, 2010; Ma *et al.*, 2014). Although nitrogen-induced soil acidification is a classical concept in soil science, the evidence recently found in China is the first empirical evidence of nitrogen-induced soil acidification anywhere in the world (Guo *et al.*, 2010). Hence, in the nitrogen recommendation system in use, the feature that nitrogen leaching could acidify soil is not taken into consideration, which increases the recommended nitrogen application rates and degrades the soil (Sheriff, 2005). In other words, the existing nitrogen control model ignores the shadow price of nitrogen, which is the value of nitrogen's effects on future soil quality and productivity (Zhang *et al.*, 2015). In contrast to the literature, this paper establishes a new nitrogen control model in a dynamic framework to study the shadow price of nitrogen, showing that ignorance of the shadow price of nitrogen explains the over-use of nitrogen and the decline in soil pH. Investigation of the effects of nitrogen control on soil acidification lies at the intersection of the classical literature on nitrogen leaching and the literature on soil acidification, which contributes to both streams of literature from the following perspectives.

In the literature on nitrogen leaching, although the environmental effects of nitrogen leaching on water quality and greenhouse gas emissions have been studied sufficiently (Kohl *et al.*, 1971; Bremner and Blackmer, 1978; Yadav, 1997; Vickner *et al.*, 1998; Shcherbak *et al.*, 2014; Kuwayama and Brozovic, 2017), the impact of nitrogen leaching on soil degradation has not been studied due to the lack of scientific evidence. The new scientific evidence on nitrogen-induced soil acidification enriches the literature by extending research on nitrogen leaching's environmental side effects to long-term loss in agricultural production. In addition, any policy implications derived from this study that mitigate soil acidification through reducing nitrogen application also potentially mitigate other non-point pollution problems associated with nitrogen leaching into the atmosphere and watersheds.

This study of nitrogen-induced soil acidification contributes to the literature on soil acidification in two ways. First, previous literature studies soil acidification as a secondary consequence of acid rain (Kaitala *et al.*, 1992; List and Mason, 2001; Li, 2014). Scientific evidence (Guo *et al.*, 2010; Ju *et al.*, 2004; Zhao *et al.*, 2010) shows that the power of nitrogen leaching on acidifying soil is about ten times that of acid rain. Thus, by introducing nitrogen leaching to the literature on soil acidification, nitrogen-induced soil acidification provides a new mechanism and essential economic intuitions with which to investigate and understand soil acidification. Second, lime applications were studied as the only approach to mitigate soil acidification (Myyra *et al.*, 2005). However, lime applications may not a feasible solution for developing countries including China.

Lime applications were compulsory investments for soil conservation during China's collective production era before the agricultural reform in the 1970s (Guo *et al.*, 2010). Due to the feature that the technology of lime application in China is labor-and-time-consuming, it became no longer cost-efficient in small-scale household production after the agricultural reform of the 1970s (Zhang *et al.*, 2016). Meanwhile, China's imperfectly protected land use rights between the 1980s and the 2000s reduced farmers' investment incentives for practicing soil conservation, which further reduced lime applications and accelerated soil acidification (Li *et al.*, 1998; Jacoby *et al.*, 2002). Hence, reversing soil acidity using nitrogen control rather than lime applications provides a new solution to soil acidification, which enlightens policy design for soil quality recovering in developing countries where lime application is not feasible due to socio-economic constraints.

To fill in the blanks in the literature, this paper establishes a discrete dynamic programming model to calculate the optimal nitrogen application associated with the potential for nitrogen-induced soil acidification. This model measures the shadow value of nitrogen, which represents the value of nitrogen's effects on future soil productivity that is not captured by the market price of nitrogen. By setting up the model in a double-crop rotation, this model captures the impact of carry-over effects of nitrogen across seasons in a rotation on soil acidification, which responds to the classical literature that a carry-over effect reduces nitrogen leaching in watersheds and the atmosphere (Stauber *et al.*, 1975; Watkins *et al.*, 1998; Ribaudo *et al.*, 2011).

Moreover, this model compares the optimal nitrogen application and soil pH at the steady states between two possible nitrogen controls: optimal nitrogen control, which takes the shadow price of nitrogen into consideration, and myopic nitrogen control, under which nitrogen investment is fully driven by the market price while ignoring the shadow price of nitrogen. My model shows that if the initial soil pH is lower than the ideal soil acidity required for the rotation, myopic nitrogen control increases the total application of nitrogen and further reduces soil acidity at the steady states. As most current nitrogen control plans do not take the shadow value of nitrogen into consideration, this model of nitrogen-induced soil acidification calculates how current myopic nitrogen control contributes to acidification, which also indicates how important it is to adjust current policies on fertilization.

Using five-year panel data of rapeseed-rice rotation based on field production experiments conducted by the Anhui Agricultural Extension Office, this paper simulates and numerically solves a dynamic programming model using the parameters estimated by the experimental data. The simulation further verifies the conclusions derived from the theoretical model. First, the shadow price of nitrogen is positive if soil pH of the entire rotation is lower than the ideal soil pH, indicating a positive value of reducing nitrogen application at the steady states and preventing further soil acidification. Second, compared with optimal nitrogen control, myopic nitrogen control applies more nitrogen in total, resulting in lower soil pH at the steady state. Third, compared with optimal nitrogen control, myopic nitrogen control applies more nitrogen in the first season but less nitrogen in the second season, which shows that the over-use of nitrogen in the first season contributes to soil acidification but the carry-over effects mitigate the acidification problem. Correspondingly, this paper provides policy implications that potentially mitigate China's soil acidification problem.

The remainder of the paper is organized as follows: section 2 presents the theoretical framework of optimal nitrogen control that takes the shadow price of nitrogen into consideration. The model is established in a double-crop rotation context, which presents the important carry-over effect across seasons in a rotation. The optimal nitrogen control model is set as the baseline for the comparison between optimal and myopic nitrogen controls. Section 3 introduces the experimental data this paper uses to estimate the parameters and simulates the dynamic programming model using the parameters estimated. This section calculates the nitrogen application and soil pH at the steady state under both optimal and myopic nitrogen control and derives the comparisons between these nitrogen control models that verify the theoretical conclusions. A series of policy suggestions are provided that could mitigate soil acidification in China. Section 4 concludes with discussion of future research directions.

Theoretical modeling and analysis

General setup

This paper sets up a discrete dynamic programming model to investigate the shadow price of nitrogen facing potential nitrogen-induced soil acidification. To generalize the model in the commonly seen rotation system and capture the importance of nitrogen carry-over (Brady and Weil, 2002), this paper sets up the model in a double-crop rotation system, allowing for intra-seasonal nitrogen allocation (Stauber *et al.*, 1975; Watkins *et al.*, 1998). It describes the dynamic program by using soil pH value (S) as the stock variable and nitrogen application in the first season (X^1) and the second season (X^2) as the control variables.

According to the nitrogen mass balance method derived from the principle of nitrogen cycle (Huang and Uri, 1993; Gilliam and Hoyt, 1987), the yield of season i in year t is a function of the absorbed nitrogen (E_t^i) and the soil pH value at the beginning of year t (S_t):

$$y_t^i = f^i\left(E_t^i, S_t\right). \quad (1)$$

The absorbed nitrogen (E_t^i), affected by an absorption coefficient (α_i), which is constant across years but varying across crops, is a proportion of the total nitrogen application. Thus, the absorbed nitrogen in the first season is $\alpha_1 X_t^1$. Regarding the nitrogen carry-over effects, the absorbed nitrogen in the second season is $\alpha_2[X_t^2+(1-\alpha_1)X_t^1]$. This model assumes that y_t^i is increasing and concave in E_t^i. Corresponding to the nature of soil such that soil pH changes slowly (Brady and Weil, 2002; Hinsinger *et al.*, 2003), this model assumes that S_t does not change within a year. Since the crop for each season has a desired soil pH value ($\overline{S}^i$) determined by its biological instinct (Brady and Weil, 2002; Zhao *et al.*, 2010), y_t^i is increasing in S_t if $S_t \leqslant \overline{S}^i$ and decreasing in S_t if $S_t > \overline{S}^i$. This model assumes that y_t^i is concave in S_t and S_t does not affect the marginal utility of absorbed nitrogen ($f_{E_t^i S_t} = 0$).

The soil pH at the end of year t (S_{t+1}) is a function of the soil pH at the beginning of year t (S_t) and a nitrogen residual (R_t). Alternative mitigation instruments, such as lime application, are excluded from the model to study the optimal nitrogen control for developing countries, where lime application is not feasible for smallholders:

$$S_{t+1} = g(S_t, R_t) \quad (2)$$

Because natural acidification of soil takes centuries, g_{S_t} is close to but no greater than 1. Thus, $g(.)$ is increasing in S_t. The nitrogen applied, if not taken up by crops, becomes the nitrogen residual (R_t) and reacts with oxygen under normal temperature and pressure conditions (Hinsinger *et al.*, 2003), acidifying soil by creating nitrates (NO_3^-) and hydrogen ions (H^+) (Hinsinger *et al.*, 2003; Guo *et al.*, 2010; Huang *et al.*, 2010). Therefore, this model assumes that $g(.)$ is decreasing in R_t. To satisfy the material balance principle (Pethig, 2006) but also simplify the mathematics, this model assumes that nitrogen influx from the atmosphere and nitrogen efflux through denitrification, volatilization and water runoff are negligible. Relaxing this assumption does not change the theoretical results of the model. Thus, the nitrogen residual after a double-crop rotation is the following:

$$R_t = (1-\alpha_2)\left[X_t^2+(1-\alpha_1)X_t^1\right]. \quad (3)$$

This model makes the assumption regarding $g_{R_t S_t}$ based on the buffering capacity of soil with respect to acidification (Brady and Weil, 2002). The buffering capacity of soil, equivalently to its resistance to acidification, is stronger as soil pH increases (Brady and Weil, 2002). Therefore, the more alkaline the soil, the more difficult to treat is the soil acidified by the nitrogen residual, indicating $g_{R_t S_t} > 0$. This model assumes that $g(.)$ is a general concave function in both arguments and that it satisfies the second-order condition.

Optimal vs myopic nitrogen control

The shadow price of nitrogen measures the value of nitrogen in long-term soil-quality maintenance, which is distinct from its values to productivity in the short run but not measured in previous nitrogen control models. Let p_t^i denote the price of the outputs in the ith season in year t, w_t denote the price of nitrogen in year t, and δ denote the discount factor. The optimal nitrogen control model maximizes the present value of crop production, which is revenue less the cost of production and the discounted land value in all future periods. Given an initial soil pH value of S_0, the optimal nitrogen control model is subject to the state equation (Equation 2). The recursion equation for the stationary problem is the following:

$$V(S_t) = p_t^1 f^1\left(\alpha_1 X_t^1, S_t\right) + p_t^2 f^2\left(\alpha_2\left(X_t^2 + (1-\alpha_1)X_t^1\right), S_t\right) - w_t\left(X_t^1 + X_t^2\right) + \delta V(S_{t+1}), \quad (4)$$

where:

$$S_{t+1} = g\left(\left[(1-\alpha_2)\left(X_t^2 + (1-\alpha_1)X_t^1\right)\right], S_t\right).$$

The first-order conditions for the interior solutions are the following:

$$\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1)p_t^2 f_{E_t^2}^2 - w_t + \sum_{\tau=1}^{\infty} \delta^\tau (1-\alpha_1)(1-\alpha_2) g_{R_t}\left[p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2\right] = 0, \quad (5)$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 - w_t + \sum_{\tau=1}^{\infty} \delta^\tau (1-\alpha_2) g_{R_t}\left[p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2\right] = 0. \quad (6)$$

In Equation (5), $\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha^1)p_t^2 f_{E_t^2}^2$ is the marginal value of applying an additional unit of nitrogen in the first season in year t. However, the additional unit of nitrogen applied in the first season in year t potentially acidifies the soil if it is not fully absorbed through the state equation, which is captured by $(1-\alpha_1)(1-\alpha_2)g_{R_t}$. Moreover, the nitrogen-induced increase in soil pH affects the marginal productivity of soil pH in all future τ years, which is captured by $p_{t+\tau}^1 f_{S_{t+\tau}}^1 + p_{t+\tau}^2 f_{S_{t+\tau}}^2$. Thus, the marginal value of applying an additional unit of nitrogen in the first season in year t equals to marginal cost of nitrogen in future productivity plus the cost of nitrogen. The same economic intuition applies to Equation (6), which equalizes the marginal cost of the nitrogen applied in the second season with the marginal value of that.

At the steady states of the optimal nitrogen control model, where $X_t^i = X_{t+1}^i$ and $S_t = S_{t+1}$, the first-order conditions are the following:

$$\alpha_1 p^1 f_{E_1}^1 + \alpha_2(1-\alpha_1)p^2 f_{E_2}^2 - w = \frac{\delta(1-\alpha_1)(1-\alpha_2)g_R\left(p^1 f_S^1 + p^2 f_S^2\right)}{\delta g_S - 1} \quad (7)$$

$$\alpha_2 p^2 f_{E_2}^2 - w = \frac{\delta(1-\alpha_2)g_R\left(p_1 f_S^1 + p_2 f_S^2\right)}{\delta g_S - 1} \quad (8)$$

$$g([(1-\alpha_2)(X_2 + (1-\alpha_1)X_1)], S) - S = 0. \quad (9)$$

Because the previous nitrogen control model ignores the impact of nitrogen residual on soil pH in the long run, the first-order conditions for myopic nitrogen control at the steady state are the following:

$$\alpha_1 p^1 f^1_{E_1} + \alpha_2(1-\alpha_1)p^2 f^2_{E_2} - w = 0, \tag{10}$$

$$\alpha_2 p^2 f^2_{E_2} - w = 0, \tag{11}$$

$$g([(1-\alpha_2)(X_2 + (1-\alpha_1)X_1)], S) - S = 0. \tag{12}$$

Comparing Equation (7) with Equation (10) and Equation (8) with Equation (11), the shadow prices of nitrogen in the first and second seasons, not captured by the myopic nitrogen control model, are the right-hand sides of Equations (7) and (8), respectively. Because $g_S \leqslant 1$ for all S, the denominators of the right-hand side in Equations (7) and (8) are negative. Therefore, the sign of the shadow price of nitrogen is consistent with the sign of $p^1 f^1_S + p^2 f^2_S$. Although the marginal productivity of soil pH could vary across seasons within one year, this paper categorizes the rotations based on the joint effects of soil pH on productivity within one year ($p^1 f^1_S + p^2 f^2_S$).

This paper defines an alkaline rotation as $p^1 f^1_S + p^2 f^2_S < 0$. In an alkaline rotation, the steady-state soil pH is higher than the desired soil pH ($\overline{S}^i$). In contrast, an acid rotation refers to a situation in which $p^1 f^1_S + p^2 f^2_S > 0$, indicating that the steady-state soil pH is lower than $\overline{S}^i$. At $p^1 f^1_S + p^2 f^2_S = 0$, the rotation is defined as a neutral rotation. The nitrogen-induced soil acidification is most harmful in the acid rotations, because the current soil pH is already below the ideal soil pH for the rotation. Although nitrogen-induced soil acidification could conceptually remove alkaline in an alkaline rotation, this approach is not recommended by soil scientists due to other properties of soil (Brady and Weil, 2002). Therefore, the following intuitive interpretation concentrates on nitrogen-induced soil acidification in acid rotations.

In the optimal nitrogen control mode, the shadow price of soil pH at the steady state (S^*) is captured by δV_{S^*}, which is $\lambda^* = \delta(p_1 f^1_{S^*} + p_2 f^2_{S^*})/1 - \delta g_{S^*}$ derived from Equations (7) and (8). Thus, in an acid rotation, the shadow price of soil pH at the steady state is positive, indicating that reducing acidity would improve soil productivity. Consistently, the shadow prices of nitrogen in both seasons, shown by the right-hand sides of Equations (7) or (8) are positive, indicating a positive loss of future soil productivity caused by nitrogen application in an acid rotation. A positive tax could be imposed on nitrogen that equals the right-hand sides of Equations (7) or (8) to discourage nitrogen application and mitigate nitrogen-induced soil acidification.

The comparison between Equation (8) and Equation (11) shows that the marginal value of the absorbed nitrogen in the second season under myopic nitrogen control (E^2_M) is greater than that under optimal nitrogen control (E^2_*). The concavity of the production function (f^2(.)) further shows that:

$$E^2_M \geqslant E^2_*,$$

$$R_M \geqslant R_*.$$

Therefore, at the steady state, with a higher nitrogen residual (R_M) under myopic nitrogen control, myopic nitrogen control contributes to soil acidification ($S_M < S_*$).

Meanwhile, $E_M \geqslant E_*$ implies that $X^2_* - X^2_M < -(1-\alpha_1)(X^1_* - X^1_M)$. Combining Equations (7) and (8), the first equation for the first-order condition is rewritten as:

$$\alpha_1 p^1 f^1_{E_1} - w = 0. \tag{13}$$

The comparison between Equations (13) and (10) shows that the nitrogen applied in the first season under optimal nitrogen control is no greater than that under myopic nitrogen control ($X^1_M > X^1_*$). This paper summarizes the theoretical results from the comparison between optimal and myopic nitrogen control as *P1*:

P1. At the steady state of an acid rotation, ignoring the shadow price of nitrogen, myopic nitrogen control leads to a higher nitrogen residual than that under optimal nitrogen control, which results in nitrogen-induced soil acidification. Over-applied nitrogen in the first season contributes directly to soil acidification regardless of the nitrogen applied in the second season.

Dynamic transitions: the policy function of soil pH

The policy function of soil pH, mapping S_t into S_{t+1}, demonstrates the dynamic transitional path of soil pH toward the steady state. The policy function of soil pH depends of the following two lemmas (for proofs see the Appendix):

Lemma 1. As the optimal soil pH increases, X^1_t and X^2_t do not decrease at the same time.

Lemma 2. For an increment in optimal soil pH value, the decrease in X^2_t is no greater than the increase in $(1-\alpha_1)X^1_t$.

Because an increase in S_t improves the resistance of soil to acidity, Lemma 1 intuitively shows that ΔS_t increases nitrogen application within at least one season. Lemma 2 further shows that an increase in S_t decreases X^2_t but increases X^1_t, indicating that it is the carry-over effect that reduces nitrogen application and thus mitigates soil acidification. Hence, Lemma 2 is consistent with the argument in previous literature (Huang and Uri, 1993) that the carry-over effect in crop rotation reduces nitrogen waste and mitigates agricultural pollution induced by nitrogen. Because of Lemma 2, $\left|-\Delta X^2_t\right| \leqslant (1-\alpha_1)\Delta X^1_t$, $\left|-\Delta X^2_t\right| \leqslant \Delta X^1_t$, indicating that the total application of nitrogen over one year $(X^1_t + X^2_t)$ increases as soil pH increases.

Lemma 1 and Lemma 2 further compares myopic and optimal nitrogen control. *P1* argues that $S_M < S_*$, and thus, because both optimal and myopic nitrogen control follow the same transitional equation, Lemma 1 shows it cannot be true that $X^1_M > X^1_*$ and $X^2_M > X^2_*$. *P1* proves that $X^1_M > X^1_*$. Therefore, it must be true that $X^2_M > X^2_*$, which indicates that the carry-over effect applies to myopic nitrogen control. Lemma 2 further shows that $X^1_M - X^1_* > (1-\alpha_1)(X^1_M - X^1_*) > \left|-(X^2_M - X^2_*)\right|$. Hence, Lemma 1 and Lemma 2 facilitate deriving the conclusion that $X^1_M + X^2_M > X_{1*} + X^2_*$, enriching the steady-state analysis. This paper summarizes these theoretical results based on Lemma 1 and Lemma 2 as *P2*:

P2. At the steady state of an acid rotation, compared with optimal nitrogen control, myopic nitrogen control applies more nitrogen in the first season but less in the second season. However, nitrogen myopically increased in the first season is greater than nitrogen decreases in the second season, resulting in higher total nitrogen application under myopic nitrogen control.

The policy function of soil pH, the mapping of S_t into S_{t+1}, depends on Lemma 1 and Lemma 2. The policy function is derived by substituting X^i_t as a function of S_t, $\Psi^i(S_t)$, in the state equation:

$$S_{t+1} = g\left(\left(1-\alpha^2\right)\left(\Psi^1(S_t) + \left(1-\alpha^1\right)\Psi^2(S_t)\right), S_t\right). \tag{14}$$

The sign of $\partial S_{t+1}/\partial S_t$ determines the slope of the policy function. Differentiating Equation (14) with respect to S_t:

$$\frac{\partial S_{t+1}}{\partial S_t} = g_{R_t}\left\{(1-\alpha^2)\left[\frac{\partial X_t^2}{\partial S_t} + (1-\alpha^1)\frac{\partial X_t^1}{\partial S_t}\right]\right\} + g_S. \tag{15}$$

Lemma 2 ensures that the sign of the brace ({.}) is positive. Still, the sign of $\partial S_{t+1\overline{\partial St}}$ is undetermined because $g_R < 0$ and $g_S > 0$. The soil pH of year t affects the soil pH in year t+1 in two ways: direct and indirect impacts. The direct impact is measured by gS_t while the indirect impact is measured $g_{Rt}\{.\}$. The direct impact captures the natural resistance of soil to acidity, the buffering capacity. The soil pH in year t in directly affects S_{t+1} through nitrogen application. According to the magnitudes of the opposite direct and indirect impacts of S_t on S_{t+1}, this paper discusses the shape of the policy function for three cases:

(1) $\left|gRt\left\{(1-\alpha^2)\left[\partial X_t^2/\partial S_t + (1-\alpha^1)\partial X_t^1/\partial S_t\right]\right\}\right| < g_S$ for all S;

(2) $\left|gRt\left\{(1-\alpha^2)\left[\partial X_t^2/\partial S_t + (1-\alpha^1)\partial X_t^1/\partial S_t\right]\right\}\right| < g_S$ for some S; and

(3) $\left|gRt\left\{(1-\alpha^2)\left[\partial X_t^2/\partial S_t + (1-\alpha^1)\partial X_t^1/\partial S_t\right]\right\}\right| > g_S$ for all S.

- Case (i): the policy function is upward sloping. The steady state is locally stable.

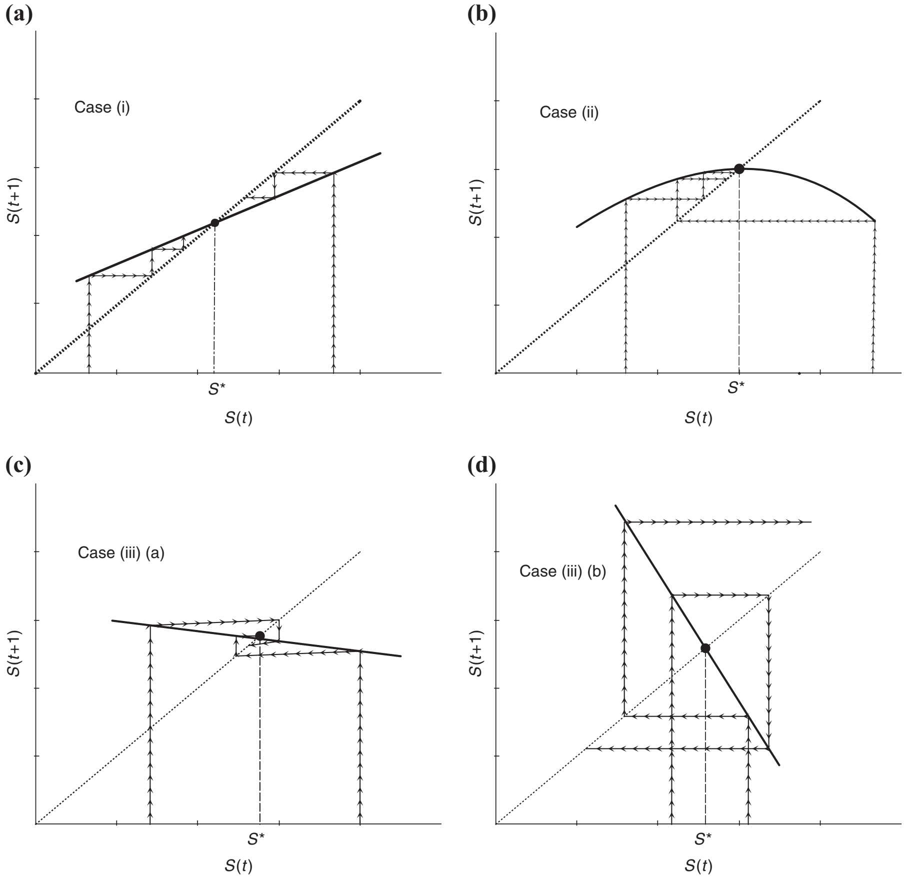
In Case (i), the magnitude of the positive direct impact is greater than that of the negative indirect impact for all S. The steady state exists and is locally stable. Because $g_S < 1$, the policy function is upward sloping with a slope flatter than the 45° line around the steady state (shown in Figure 1(a)). Intuitively, if the magnitude of soil-buffering capacity with respect to acidification is stronger than the indirect impact on S_{t+1} through nitrogen application, although nitrogen application acidifies soil if $S_t > S_*$, soil acidification happens slowly as the soil pH moves toward the steady-state soil pH (S_*) The strong buffering capacity prevents rapid and severe soil acidification. Case (i) represents the dynamic transition of soil pH in places where soil is not sensitive to acidity:

- Case (ii): the policy function is inverse-U shaped, reaching its peak at the steady state. The steady state is locally stable.

In Case (ii), the magnitude of the direct impact is greater than that of the indirect impacts for some possible S. Because g_S is decreasing in S, the possibility of a "*V*"-shaped policy function is ruled out. The steady state at the peak of the inverse-*U* shaped policy function is locally stable (shown in Figure 1(b)). The reason that the policy function goes down if $S_t > S_*$ is that the negative indirect impact through nitrogen offsets the positive direct impact through the buffering effect as S_t goes beyond the steady state. Regarding the dynamic path, the difference between Case (i) and Case (ii) is that soil pH declines rapidly toward a soil pH that is even lower than S_* if $S_t > S_*$ in Case (ii). Thus, Case (ii) describes the dynamic transition of soil pH with comparatively weaker buffering capacity. The problem of nitrogen-induced soil acidification affects the soil pH more severely in Case (ii) than that in Case (i):

- Case (iii): the policy function is downward sloping. The stability of the steady state depends on the slope of the policy function.

In Case (iii), the indirect impact is greater than the direct impact for all S. The stability of the steady state depends on the slope. A flatter slope $\left(\left|\partial S_{t+1}/\partial S_t\right| \leqslant 1\right)$, as shown in Figure 1(c), creates a stable steady state, ensuring that the soil pH moves toward the



Notes: (a) Policy function of case (i); (b) policy function of case (ii); (c) policy function of case (iii-a); (d) policy function of case (iii-b)

Figure 1. Policy functions of Cases (i)–(iii)

steady state soil pH spirally. With an unstable steady state under a steeper slope ($|\partial S_{t+1}/\partial S_t| > 1$), the soil pH moves away from the steady state. With an unstable steady state, soil pH at any value could experience severe soil pH. Even for a stable steady state, soil pH takes a longer pathway than in the situations of Case (i) and Case (ii) to move toward the steady state, jumping between a soil pH value higher than S_* and a soil pH value that is lower than S_* several times with rapid and huge changes in soil pH. Therefore, the weak buffering capacity contributes to the most severe nitrogen-induced soil acidification.

Simulation analysis

This paper conducts a simulation exercise to numerically solve optimal and myopic nitrogen control models at the steady state to verify the theoretical results in *P1* and *P2* using farming experimental data from rapeseed-rice rotation provided by the Anhui Agricultural Extension Office (Qian, 2010). The simulation analysis includes two steps: estimating the production function and the state equation and numerically solving optimal and myopic dynamic programming models based on the theoretical framework. The empirical hypotheses derived from *P 1* and *P2* are listed as follows:

H1. In an acid rotation, the shadow price of nitrogen is positive, indicating a positive value of nitrogen in future production through soil pH maintenance.

H2. In an acid rotation (the sum of the marginal values of soil pH in two seasons is positive), compared with optimal nitrogen control, myopic nitrogen control applies more nitrogen in total, resulting in higher soil pH.

H3. In an acid rotation, compared with the optimal nitrogen control, a myopic nitrogen control applies more nitrogen in the first season but less in the second season.

Data description

This paper uses experimental data on rapeseed-rice production in Anhui Province to estimate the production function and state equation of soil pH. The experimental data cover all the national long-term nine monitoring fields in Anhui Province in experiments conducted from 2005 though 2010, which is also a subsample of the data used in Guo *et al.* (2010). Anhui Province is located in central China and encompasses all three types of terrain (plains, hills, and mountains) and all five types of soil. Thus, experimental data collected in Anhui province are representative of agricultural production nationwide.

The experimental data populate a five-year panel data set on 36 plots. Rapeseed rotation is a common practice that occupies 70% of farmland in Anhui Province (Wang and Luo, 2006). The first season, for rapeseed production, runs from mid-October to late May. The second season, for rice production, runs from late May to early October of the next year. The order of the rotation is determined by climate related factors. The data are taken from records of nitrogen applications and yields for each season and the soil pH at the end of an entire rotation. Nitrogen in each season is applied at six possible levels, ranging from 5 to 10 kg/mu, 10 kg/mu to 15 kg/mu, 15 kg/mu to 20 kg/mu, 20 kg/mu to 25 kg/mu and 25 kg/mu to 30 kg/mu. At each possible nitrogen level, on six plots the same levels of nitrogen were applied in each season. With varying combinations of nitrogen levels over two seasons, the experiment included 36 types of nitrogen application plans. Rapeseed yields ranged from 269 kg/mu to 432 kg/mu and rice yield ranged from 450 kg/mu to 1,392 kg/mu. Data on soil properties were recorded annually at the end of the second season. Soil pH varied from 4.6 to 8.4, which covers weakly acidic to weakly alkaline soil pH (Brady and Weil, 2002).

Parameter estimation

The production functions for crops and the state equation of soil pH are estimated in a reduced form with arguments X_t^i and S_t Being consistent with the assumptions of the theoretical model, this paper imposes quadratic functional forms for production functions and the state equation:

$$f^1\left(X_t^1, S_t\right) = a_0^1 + a_1^1 X_t^1 + a_2^1\left(X_t^1\right)^2 + a_3^1 S_t + a_4^1 (S_t)^2 + a_5^1 S_t X_t^1 + e_t^1, \tag{16}$$

$$f^2\left(X_t^2, S_t\right) = a_0^2 + a_1^2 X_t^2 + a_2^2\left(X_t^2\right)^2 + a_3^2 S_t + a_4^2 (S_t)^2 + a_5^2 S_t X_t^2 + a_6^2 X_t^1 + e_t^2. \tag{17}$$

The error terms,e_t^1 and e_t^2, are independent independent and identically distributed for each plot. The production function also allows for interactions between nitrogen application X and soil pH S. Based on the assumptions associated with the state transition equation (Equation 2), the estimated state equation is quadric in S_t and linear in X_t:

$$S_{t+1} = k_0 + k_1 S_t + k_2 (S_t)^2 + k_3 X_t^1 + k_4 X_t^2 + k_5 S_t X_t^1 + k_6 S_t X_t^2 + \varepsilon_t. \tag{18}$$

In Equation (18), ε_t is an independent and identically distributed error term. Soil pH at the end of the rotation (S_{t+1}) is determined by nitrogen applications in both seasons and the interaction between nitrogen applications and *St*.

Applying plot fixed effects and year fixed effects, this paper estimates Equations (16)-(18) ordinary least squares by OLS. The results are shown in Table I. Due to a limited number of observations, this paper applies Monte Carlo to test the applicability of the OLS approach. The OLS results shown in Table I are similar to the results derived from Monte Carlo shown in Table II. Comparing the results shown in Table I with those shown in Table II, the OLS estimates are close to the mean of the Monte Carlo coefficients. Thus, the OLS results are unbiased and reliable for further empirical analysis.

In rapeseed production, all the coefficients are jointly significant at the 0.01 significance level. The coefficients associated with X_t^1, $(X_t^1)^2$, and $X_t^1 S_t$ are jointly significant at the 0.05 significance level. The coefficients associated with S_t and S_t^2 are both significant at a 0.01 significance level. Plugging in the range of nitrogen applications, the desired range of soil

	Rapeseed (y_t^1)	Rice (y_t^2)	State equation (S_t+1)
X_t^1	21.07 (39.90)	9.17 (8.92)	0.39** (0.16)
$(X_t^1)^2$	−1.34** (0.53)		
X_t^2		71.98 (50.01)	0.03 (0.29)
$(X_t^2)^2$		−4.11** (1.32)	
S_t	413.52*** (71.04)	31.09 (88.33)	2.09*** (0.42)
S_t^2	−26.25*** (4.99)	−6.19 (9.95)	−0.03 (0.07)
$S_tX_t^1$	2.04 (5.42)		−0.07 (0.03)
$S_tX_t^2$		3.31 (7.82)	−0.00005 (2.00)
No. of Observation	180	180	180
R^2	0.32	0.41	0.80
Year fixed effects	*Y*	*Y*	*Y*
Plot fixed effects	*Y*	*Y*	*Y*
F-stat. of non-constant terms	77.55***	14.27***	23.72***
Chi-stat. of X_t^i related terms	11.58**	2.79**	4.59**
Chi-stat. of S_t related terms	161.68***	554.22***	23.73***

Notes: Robust SE clustered at the village level in parentheses. *p < 0.1; **p < 0.05; ***p < 0.01

Table I. Estimation of crop production functions and state equation

	Rapeseed (y_t^1)	Rice (y_t^2)
X_t^1	20.62 (16.92)	9.13*** (3.37)
$(X_t^1)^2$	−1.32** (0.65)	
X_t^2		74.26** (34.30)
$(X_t^2)^2$		−4.21* (2.40)
St	411.47*** (103.50)	34.52 (99.47)
S_t^2	−25.28 (23.88)	−6.40 (24.97)
$S_tX_t^1$	1.94 (13.76)	
$S_tX_t^2$		2.96
(18.63)	(6.70)	
No. of Observation	180	180
Year fixed effects	*Y*	*Y*
Plot fixed effects	*Y*	*Y*

Notes: Robust SE clustered at the village level in parentheses. *p < 0.1; **p < 0.05; ***p < 0.01

Table II. Monte Carlo simulation of crop production pH ($\overline{S^{rapeseed}}$) is between 8.0 and 8.8. Rapeseed yields increase with soil pH if $S_t \leqslant \overline{S^{rapeseed}}$ and decrease if $S_t > \overline{S^{rapeseed}}$. The desired range of soil pH for rapeseed production is between 8.03 and 8.79, as determined by plugging in the range of nitrogen applications for all rapeseed observations. Rapeseed yield increases with soil pH if the soil pH is less than $\overline{S^{rapeseed}}$. According to the parameter estimation, the rapeseed production function is increasing and concave in S and X_t^1 because the quadratic terms are associated with negative coefficients. Rapeseed yield reaches its peak at $X_t^1 = 12.8$ kg/mu.

In rice production, all the coefficients' parameters are jointly significant at the 0.01 significance level. The coefficients associated with X_t^2, $(X_t^2)^2$ and $X_t^2 S_t$ are jointly significant at the 0.05 significance level. The coefficients associated with X_t^1, X_t^2 and $(X_t^2)^2$ are jointly significant at the 0.01 significance level, showing the importance of the carry-over effect from the previous season. The coefficients associated with S_t and its quadratic term are jointly significant at the 0.05 significance level. The rice yield increases with soil pH if the soil pH is less than the desired soil pH for rice ($\overline{S^{rice}}$). The estimated $\overline{S^{rice}}$ is between 5.2 and 6.41. Rice production reaches its peak at $X_t^2 = 12.4$ kg/mu. Consistent with assumptions mentioned in the theoretical considerations, the rice production function is concave and increasing with nitrogen and soil pH.

Consistent with the assumption, the state equation is concave with soil pH because the coefficient associated with $(S_t)^2$ is negative. The marginal product of soil pH (f_{S_t}) is less than 1 within the range of nitrogen application in the data set. Nitrogen applications X_t^1 and X_t^2 are jointly significant at the 0.01 significance level. The interaction terms for nitrogen and soil pH are jointly significant at the 0.01 significance level. Within the range of soil pH of the data set, an additional kilogram of nitrogen applied in the first season decreases the soil pH by at least 0.03, while an additional kilogram of nitrogen applied in the second season increases the soil pH by no more than 0.02. The estimation of the state equation shows that the carry-over effects maintain soil acidity (Huang and Uri, 1993).

Numerical solutions at the steady state

The steady states are characterized by Equations (7)–(9). The specific function forms with the estimated parameters are used to calculate the steady states of the optimal nitrogen control in a rapeseed-rice rotation. The parameters that are not significant in the estimations of production functions and the state equations are not eliminated because all coefficients are jointly significant. The average transaction price of pure chemical nitrogen between 2011 and 2014 was 2.2 RMB/kg (FERT, 2014). Between 2011 and 2014, the average farm-gate price of rapeseed was 4.5 RMB/kg (NRDC, 2011a) and the average farm-game price of rice was 2.4 RMB/kg (NRDC, 2011b). In the following analysis of steady states, δ is set at 0.9. The numerical solutions are reported in Table III.

At the steady state, the optimal nitrogen application for rapeseed is 13.29 kg/mu and the optimal nitrogen application for rice is 12.06 kg/mu. The steady-state soil pH is 7.86, which is weakly alkaline. The shadow price of nitrogen in rapeseed production is positive at 26.27 RMB, which is 4.8 times greater than its market price. The shadow price of nitrogen in rice production is 4.92 RMB, which almost doubles its market price. Based on the theoretical results, the positive shadow price of nitrogen indicates that nitrogen, if not applied, adds

Table III. Optimal vs myopic nitrogen management at the steady state

	X^1	X^2	S	X^1+X^2
Optimal nitrogen control	13.29 kg/mu	12.06 kg/mu	7.86	25.35 kg/mu
Shadow price of nitrogen	26.26 RMB	4.92 RMB		
Myopic nitrogen control	15.90 kg/mu	11.50 kg/mu	7.09	27.4 kg/mu

positive value to production in the future, which mitigates soil acidification caused by nitrogen leaching. Thus, *H1* is successfully tested.

The rapeseed-rice rotation is an acid rotation with a positive $p^1 f_S^1 + p^2 f_S^2$. The steady state under myopic nitrogen control is calculated by setting the shadow price of nitrogen at 0. The myopic nitrogen applied in the first season is 15.90 kg/mu, which is greater than the optimal nitrogen application at 13.29 kg/mu. Myopic nitrogen applied in the second season is 11.50 kg/mu, which is lower than the optimal nitrogen application at 12.06 kg/mu. The total nitrogen applied under myopic nitrogen control is 27.40 kg/mu, which is higher than the total nitrogen applied in the optimal nitrogen control at 25.35 kg/mu. Under myopic nitrogen control, the steady-state soil pH is 7.09, which is more acidic than the steady state under optimal nitrogen control at 7.86. Thus, this paper empirically verifies both *H1* and *H2*.

Policy implications

While most of China's current fertilization policies are designed to encourage the use of nitrogen fertilizer, the shadow price of nitrogen is not fully considered in the nitrogen recommendation system designed by the agricultural extension office. Therefore, the current recommended nitrogen control plans ignore the shadow price of nitrogen and are therefore equivalent to myopic nitrogen control, which induces the problem of soil acidification. Based on the simulation results of a rapeseed-rice rotation, a representative acid rotation, this paper empirically tests the theoretical framework and derives three policy implications for addressing the problem of nitrogen-induced soil acidification in China. As nitrogen fertilizer is unconditionally encouraged in agricultural production in Africa (Lunduka *et al.*, 2013; Jayne *et al.*, 2013) and lime applications are also not feasible in Africa, the lessons for soil acidification learned by China and the potential for using nitrogen control to mitigate soil acidification could be enlightening for policy design in African countries.

First, agricultural extension offices could establish education programs that focus on increasing awareness of the shadow price of nitrogen in long-term agricultural production (Pan *et al.*, 2017). The comparison between optimal and myopic nitrogen control shows that erroneous instructions for nitrogen control decrease soil pH from 7.86 to 7.09. Even a 0.5 decrease in soil pH increases the amount of dissolved toxic elements 1,000 times (Bolan and Hedley, 2003; Brady and Weil, 2002), which greatly but negatively affects agricultural production (Brady and Weil, 2002). Thus, an education program that guides nitrogen control scientifically and sustainably by considering the shadow price of nitrogen could help correct the distorted investment incentives in nitrogen and mitigate soil acidification.

Second, both the theoretical and empirical results show that although total nitrogen applied is important in affecting soil acidity, the allocation of nitrogen across seasons plays an even more important role in the process of soil acidification. In the simulation of a rapeseed-rice rotation, although myopic nitrogen control applies 2.05 kg/mu more nitrogen than optimal nitrogen control over the entire rotation, the difference in soil pH is huge. The shadow price of the first season is more than five times of that of the second season. Thus, even while keeping the current total nitrogen application the same, applying less nitrogen in the first season mitigates nitrogen-induced soil acidification, which reduces the total loss associated with the over-applied nitrogen in total. Therefore, if agricultural extension offices could provide precise nitrogen recommendations for each season, nitrogen-induced soil acidification could be mitigated.

Third, a regular taxation policy might be unenforceable because the shadow prices of nitrogen differ in the two seasons (Martinez and Albiac, 2006). In a rapeseed-rice rotation, a tax of 26.26 RMB/kg should be added to nitrogen applied in the first season and a tax of 4.92 RMB/kg should be added to nitrogen applied in the second season. However, because nitrogen fertilizer is storable, a greater purchase in the second season that avoids the high tax for nitrogen in the first season could lead to nitrogen application in the first season that is higher than the optimal application rate. In addition, a one-time nitrogen purchase within a year is time-saving for farmers. A taxation policy that requires two purchases in one year is not practical for regulating nitrogen applications. Therefore, on the one hand, crop rotation reduces soil acidity through the carry-over effects while, on the other hand, such rotation makes it challenging to provide the right economic incentives that encourage farmers to benefit from the implications of optimal nitrogen control.

Conclusion

The problem of soil acidification has concerned agricultural economists for ages. However, the soil acidification problem has not been structured under the framework of nitrogen control without lime application. Meanwhile, nitrogen leaching is also a classical problem in agricultural pollution. However, the pollution caused by nitrogen leaching into soil and thereby harming soil quality has not been studied. Relying on the most up-to-date scientific research, this paper establishes a new dynamic framework for nitrogen control to solve for optimal nitrogen applications in a double-crop rotation that considers the effects of nitrogen on soil acidity.

Theoretically, this paper measures the shadow price of nitrogen. Then, the paper concludes that the shadow price of nitrogen is positive if the soil pH of the entire rotation is lower than the ideal soil pH (an acid rotation), which prevents further nitrogen investment that would lead to more serious soil acidification. Comparing optimal nitrogen control with a non-negative shadow price of nitrogen with myopic nitrogen control, this study finds that myopic nitrogen control applies more nitrogen in total but with a higher nitrogen application in the first season and a lower nitrogen application in the second season. Thus, it is the over-use of nitrogen in the first season that contributes to soil acidification with myopic nitrogen control.

The theoretical results correspond to three testable empirical hypotheses. Using experimental data from an agricultural extension office, this paper numerically solves dynamic programming model for both optimal and myopic nitrogen controls. In addition to verifying the theoretical results by comparing these two types of nitrogen control at the steady state, this paper calculates the shadow price of nitrogen for each season and derives three policy implications that facilitate future policy design in mitigating nitrogen-induced soil acidification.

Although ignoring the shadow price of nitrogen in nitrogen control explains the decline in soil pH and over-use of nitrogen, the application rate of nitrogen in some regions is even higher than the application rate guided by myopic nitrogen control, which results in a further decline in soil pH. However, this phenomenon could not be fully explained by the theoretical framework of this paper. Thus, further theoretical and empirical research could focus on the impacts of other socio-economic factors on nitrogen application and institutional constraints on the feasibility of lime applications. Specifically, the trade-off between a precautionary action (reducing the amount of nitrogen applied) and a preventative action (lime application) should be more rigorously studied within a dynamic framework. Meanwhile, changes in the Chinese rural land property rights system over the past several decades and the process of labor-switching from the agricultural sector to non-agricultural sectors are also major factors that affect investment incentives in nitrogen and lime applications, which are worth studying in the future with a panel of fixed-point household data with information on agricultural production and soil quality.

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Appendix

Proof of Lemma 1

In the social optimal farm management, the first-order conditions for interior maximum are:

$$\alpha_1 p_t^1 f_{E_t^1}^1 + \alpha_2(1-\alpha_1) p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_1)(1-\alpha_2) g_{R_t} V_{S_{t+1}} = 0, \tag{A1}$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 - w_t + \delta(1-\alpha_2) g_{R_t} V_{S_{t+1}} = 0. \tag{A2}$$

Lemma 1 is proved by contradiction in three steps as shown below.

Step 1: prove that $\alpha_2 p_t^2 f_{E_t^2}^2$ is greater in $X^1(S^0)$, $X^2(S^0)$, and S^0.

Suppose there exists $S^* < S^0$, such that $X^2(S^*) > X^2(S^0)$ and $X^1(S^*) > X^1(S^0)$, by having $f_{ES} > 0$ and the concavity of $f(.)$ in E and S, I derive:

$$\alpha_2 p_t^2 f_{E_t^2}^2 \left(X^1\left(S^0\right), X^2\left(S^0\right), S^0\right) > \alpha_2 p_t^2 f_{E_t^2}^2 \left(X^1\left(S^0\right), X^2\left(S^0\right), S^*\right)$$

$$> \alpha_2 p_t^2 f_{E_t^2}^2 \left(X^1(S^*), X^2(S^*), S^*\right). \tag{A3}$$

Step 2: prove that $\delta(1-\alpha_2) g_{R_t} V_{S_{t+1}}$ is greater in $X^1(S^0)$, $X^2(S^0)$, and S^0.

Since $X^2(S^*) > X^2(S^0)$ and $X^1(S^*) > X^1(S^0)$, I get $R_t^0 < R_t^*$. Because $g(.)$ is concave in R, I have $g_{R_t^0} > g_{R_t^*}$. Since $g(.)$ is super modular in R and S, $g(R_t^0, S_0) > g(R_t^*, S_0) > g(R_t^*, S_*)$. In other words, $S_{t+1}^0 > S_{t+1}^*$. Again, the concavity of value function makes $V_{S_{t+1}^0} < V_{S_{t+1}^*}$. Thus:

$$\delta(1-\alpha_2) g_{R_t^0} V_{S_{t+1}^0} > \delta(1-\alpha_2) g_{R_t^*} V_{S_{t+1}^*}. \tag{A4}$$

Step 3: prove that Equation (A2) is greater in $X^1(S^0)$, $X^2(S^0)$ and S^0.

Adding Equations (A3) to (A4):

$$\alpha_2 p_t^2 f_{E_t^2}^2 \left(X^1\left(S^0\right), X^2\left(S^0\right), S^0\right) + \delta(1-\alpha_2) g_{R_t^0} V_{S_{t+1}^0} >$$

$$\alpha_2 p_t^2 f_{E_t^2}^2 \left(X^1(S^*), X^2(S^*), S^*\right) + \delta(1-\alpha_2) g_{R_t^*} V_{S_{t+1}^*} = 0. \tag{A5}$$

Here, in Equation (A5), there is a contradiction because as $X^1(S^0)$ and $X^2(S^0)$ maximize the value function when soil pH is S^0, $X^1(S^*)$ and $X^2(S^*)$ also maximize the value function when soil pH is S^*. The contradiction is derived by comparing Equation (A5) with Equation (A2). Thus, I arrive at Lemma 1. As soil pH goes up, the optimal nitrogen application of seasons 1 and 2 do not decrease at the same time. Intuitively, since nitrogen induces soil acidification, the higher the soil pH value, the more tolerate the soil is to acidification. Thus, absolute decrease of nitrogen in both seasons is not reasonable.

Proof of Lemma 2

After knowing how nitrogen in seasons 1 and 2 influenced by soil pH, it is the right time to know exactly how nitrogen in seasons 1 and 2 interact. A similar contradiction is constructed by using Equation (A1):

Suppose there exists $X_{1t}^* > X_{1t}^0$, such that $X_{2t}^* > X_{2t}^0$; in this case, I arrive at the similar contradictory condition that (compared with Equation (A1)):

$$\alpha_1 p_t^1 f_{E_{1t}}^1 \left(E_{1t}^0, S_t\right) + \alpha_2(1-\alpha_1) p_t^2 f_{E_{2t}^0}^2 \left(E_{2t}^0, S_t\right) + \delta(1-\alpha_2) g_{R_t^0} V_{S_{t+1}^0} >$$

$$\alpha_1 p_t^1 f_{E_{1t}^*}^1 \left(E_{1t}^*, S_t\right) + \alpha_2\left(1-\alpha^1\right) p_t^2 f_{E_{2t}^*}^2 \left(E_{2t}^*, S_t\right) + \delta(1-\alpha_2) g_{R_t^*} V_{S_{t+1}^*} = 0. \tag{A6}$$

Thus, I prove that when X_t^1 increases at the optimal, X_t^2 does not increase. When optimal nitrogen application in season 1 increases, keeping the absorption coefficient constant, the nitrogen residual from season 1 increases, as well. The residual is involved in the crop production in season 2. Thus, without increasing nitrogen application in season 2, the availability of nitrogen in season 2 improves. For any level of soil pH, if increase in X_t^1 increases X_t^2, then the residual after the entire rotation goes up. Soil pH cannot stay at the same level if residual after season 2 goes up via the state equation. Thus, for optimal management, at any level of soil pH, carry-over effect of season 2 reduces the application of nitrogen. I summarize this result as:

Lemma 2. For any levels of soil pH value, any increment of optimal nitrogen in season 1 decreases optimal nitrogen in season 2.

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