



Endowments, Technology Choices, and Structural Change*

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Abstract

Modern economic growth features continuous structural change in technology and industry. This paper is motivated by the empirical facts that technology in each industry, as well as the whole economy, becomes more capital intensive and the industrial structure is unimodal, with a leading industry at any time in the process of economic development. We set up an infinite-industry general equilibrium model to discuss technology choice and structural change in economic development. The model shows that along with the increase in capital endowment, every industry experiences technology upgrading from labor intensive to capital intensive, the industrial structure shifts more and more to capital intensive industries, and the leading industry is taken over by a new leading industry continuously. The predictions of the model are tested with a data set from the National Bureau of Economic Research–U.S. Census Bureau's Center for Economic Studies, and a counterfactual simulation is used to test the implications of the model. In a dynamic extension of the model, a modified Keynes-Ramsey rule is presented. The model provides a workhorse for multi-industry structural analysis.

Keywords: Technology Choice, General Equilibrium, Structural Change, Endowment.

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1 Introduction

The way a firm applies technology to combine various inputs into output, reflected in a production function, is one of the most fundamental issues in the economic research. It is a common practice in economic research to assume the production function to be exogenous and given, no matter whether the economy to be modelled is one sector or multi-sectors. The given production function implies that firms only choose the quantity of input, but not the way to combine them. This assumption also implies that countries in different level of development only has quantitative differences rather than qualitative differences. However, firms that produce the same products have choices over a technology set. One example is the adoption of robots in producing manufacturing goods. Figure 1 shows the robot intensity, i.e. the number of installed industrial robots per 10,000 employees in manufacturing industry of different countries in 2017, surveyed by the International Federation of Robotics ¹. As we can see from the graph, the robot intensity has large variation across different countries. The report also documents that China has the fastest development in robot density from 25 units in 2013 to 68 units in 2016 ². This phenomenon indicates dramatic differences in technologies across countries.

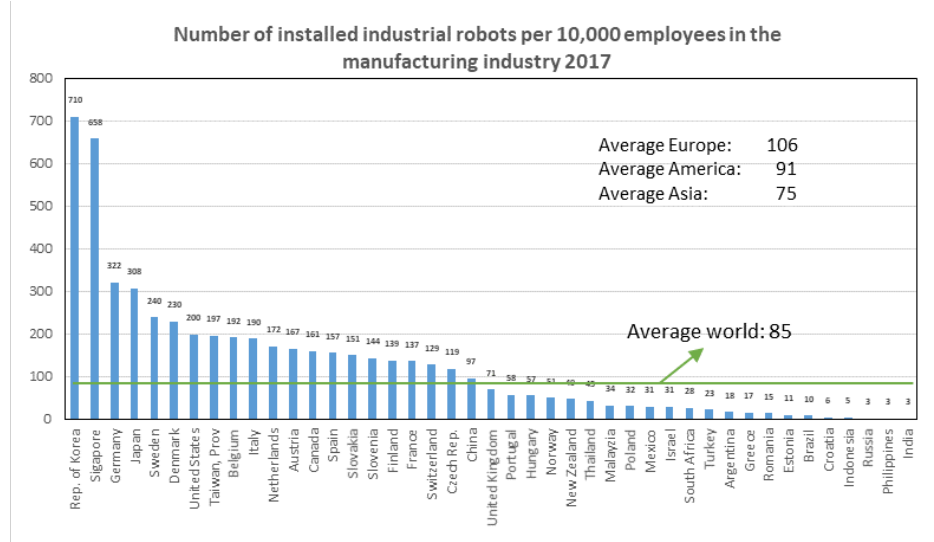


Figure 1: Robot Intensity in 2016

The technology changes could have different directions. On one hand, we constantly observe manufacturing industries utilize more and more machines and automation equipment, i.e. to use increasingly capital-augmenting / capital-complementary / capital-biased technologies. On the other hand, human capital improvement enables firms to use more labor-augmenting / labor-complementary / labor-biased technologies. The changing technologies appear in the form of evolving capital-labor ratio in the same industry. We show later in Section 2 that the aggregate capital-labor ratio of US manufacturing industries increased constantly over the period from 1958 to 2011. More importantly, the increasing capital intensity was persistent within each disaggregated industry. These observations

¹<https://ifr.org/ase-studies/industrial-robots/four-yaskawa-motoman-handling-robots-feed-two-turning-machines-for-the-manu>

²<https://interestingengineering.com/global-robot-density-rose-to-74-robots-per-10000-workers-in-2016>

imply that production technology is changing over time and the production function is evolving due to firms' endogenous choice of production technology, rather than an exogenously given function.

The literature of appropriate technology addresses this issue. The tradition goes back to Atkinson et al (1969), and was later modeled in Diwan and Rodrik (1989), Basu and Weil (1998), and Acemoglu and Zilibotti (2001), arguing that countries with different endowments (and thus different factor prices) should choose different technologies. One influential paper, Caselli and Coleman (2006) builds a one-sector model of technology choices to study the imperfect substitution between skilled labor augmenting and unskilled labor augmenting technologies. The paper studies cross-country differences in endowment, technology choices and factor prices (skill premium). The positive correlation between skill premium and skill labor endowment across countries is explained by imperfect substitutes between unskilled and skilled labor, as well as differences in their technology frontiers. Similarly, Leon-Ledesma and Satchi (2016) study the appropriate technology in a one-sector growth model.

The use of a single sector economy in the existing literature has its limitations. By assuming the economy as one sector, these models overlooked the impacts of evolving industrial structure on technology choices. Imagine a one-sector economy, where we observe increasing endowment of skill labor, along with increasing skill premium over time. The approach in Caselli and Coleman (2006) would infer great improvement of technology frontiers along the dimension of skill-labor augmenting factor, since the return to skill labor does not decrease with an increase in the supply of skill labor. However, if we consider evolving industrial structure from un-skill labor intensive to skill labor intensive industries, the expansion of skill labor intensive industries raises the demand for skill labor and drives up the skill labor return. Therefore, the inferred changes in technology frontiers would not exist or be smaller than under the one-sector assumption. This thought experiment illustrates the necessity of multi-industry setup when discussing economic development.

In this paper, we discuss the technology choices in an economy with infinite industries, and relate the technology choices with endowment-driven structural change across industries (Ju, Lin and Wang 2015). We contribute to the literature by answering the following questions: How do we explain the endogenously evolving capital intensity of each industry? While the aggregate economy becomes more capital abundant, how do the technology choices of each industry change? Are industries using capital more efficiently, or using labor more efficiently? How are capital intensities, technology choices and industrial market shares related? Summing up, we explore how the endowments shapes technology choices and structural change of industries in an economy.

Before theoretical analysis, we first investigate capital intensities in each industry by exploring NBER-CES dataset of US manufacturing industries during the period between 1958 and 2010. We find a strong pattern that capital intensity (defined as the ratio of capital over labor or capital income share) in each industry, as well as in the economy, is increasing over time. We then extend Caselli and Coleman (2006) model to a multi-industry (infinite industries) economy in the general equilibrium framework. In the model, technology choices are modelled as the augmenting factors for capital and labor in a CES function. The two augmenting factors are bounded by technology frontier: technologies that are more efficient in using labor are less efficient in using capital, and vice-versa. We find that with an increase in capital endowment, rental-wage ratio declines and economic structure moves from labor-intensive to capital-intensive industries. In addition, capital intensities of each industry increase. The technology choices in each industry depend on aggregate endowment, technology frontier, as well as the elasticity of substitution between capital and labor. When the elasticity of substitution between

capital and labor is lower / higher than one, the accumulation in total capital endowment leads to lower / higher capital-augmenting technology, and higher / lower labor-augmenting technology.

When fixed technology frontier exists in each industry, there is a trade-off between capital improving and labor improving technologies. Intuitively, when capital endowment becomes more abundant, real rental price (defined as rental price over wage) becomes lower. If capital and labor are substitutes, cheaper capital price encourages more intense usage of capital. On the contrary, if capital and labor are complementary, more abundant capital implies more scarce labor endowment. As a result, one must adopt the “labor-augmenting” technology to increase the efficient labor input. Therefore, the technology choice between capital and labor depends crucially on the elasticity of substitution between capital and labor.

After setting up the model, we compute technology choices of disaggregated industries for the US manufacturing sector. By applying Caselli and Coleman (2006) to multi-industries, we compute annual technology choices as augmenting factor in a CES production function for each industry, and estimate the technology frontiers of each industry in every year. Using the quantitative model, we conduct one counterfactual test and show that, without the technology choices, the structural change from labor intensive to capital intensive sectors would become slower.

The paper is related to literature on the following branches. Firstly, we extend the literature on Appropriate technology (Caselli and Coleman 2006, Leon-Ledesma and Satchi 2011, 2016) in one-sector economy to multi-sector economy. By doing so, our model shows that changes in industrial structures are closely related with technology choices, and our empirical analysis confirms the model predictions. There is also a growing literature on directed technology change (Kennedy (1964), Samuelson (1965), Drandakis and Phelps (1965), David (1975), Acemoglu (1998, 2002, 2007)). This literature emphasizes factor-biased, rather than aggregate technical change. The total endowment affects the technical change through price effect and market size effect. Our departure from literature is that we consider multi-sector rather than one sector economy, so as to relate the technical changes with industrial structure. We jointly explain the changes in technology choices, capital intensity and market share of each industry. In addition, we show the longrun changes in technology choices and technological frontiers in the US empirically. We also provide an explanation for the general observations of capital deepening (Acemoglu and Guerrieri 2008) and declining labor share (Karabarbounis and Neiman 2014). We contribute to the literature by showing declining labor share within each industry, due to change in endowment and endogenous technology choices.

Secondly, our paper discusses endowment-driven structural change. Despite that the observation of structural change has been documented in earlier research (such as Matsuyama 2008, Herrendorf, Rogerson and Valentinyi 2011, Kongsamut, Rebelo, and Xie 2001, Caselli and Coleman 2001, Wang and Xie 2004, etc), we are the first to construct a theoretical model to jointly explain structural change with technology choices. In the Solow model, an economy with rapid accumulation of capital should experience a fast decline in capital return (Solow 1955), which was not found in the Asian Tigers and China during their fast economy growth (Lin and Ren (2007), Song, Storesletten and Zilibotti (2011)). This paper provides an explanation: the changing industrial structure from labor intensive to capital intensive sectors provides a large demand for capital and drives up the return of capital.

The rest of paper is organized as follows. In section 2, we introduce three stylized facts about observed capital intensity. Section 3 constructs the model and derives testable predictions. Section 4 computes the technology choices based on the model and presents general findings about industrial

structural and technology choices. Section 5 shows a simulation of the model and counterfactual analysis of fixed technology frontiers. Section 6 shows an alternative approach to model technology choices in different industries. Section 7 constructs a dynamic growth model with technology choices. Section 8 concludes.

2 Stylized Facts of Capital Shares

We begin our analysis by investigating capital intensities for disaggregated US industries. The observed capital intensity contains information about unobserved technology choice of each industry. Take a simple example, if we assume production function to be a Cobb-Douglas production function $y = AK^\alpha L^{1-\alpha}$. In this function, α measures technology choices. Changes in capital share (and hence capital-labor ratio) indicates changes in technology choices. Later we show in our model that capital intensity also reflects changes in technology choices when we assume a general CES production function.

We use the NBER-CES data set of the US manufacturing sector, which covers 495 industries at the 4-digit ISIC level from 1958 to 2011. Supplementary data of the UNIDO data set is also used, which covers 148 countries and 18 manufacturing sectors from 1963 to 2014. We measure capital intensity by capital income share, i.e. one minus the share of wage income in value added. We compute the capital share for each industry in each year using NBER-CES dataset for US, and that for each industry in each country, each year using UNIDO dataset. We lay out the following three key findings.

Finding 1: Aggregate capital share is increasing.

Figure 2 shows the change in aggregate capital income share for US over time. We first compute the total capital stock K , employment L , wage payment WL and value added VA , then define the aggregate capital intensity as the capital income share $(VA - WL)/VA$. We see that the capital income kept increasing over the period from 1958 to 2011. Although well documented by the recent literature on declining labor share (Karabarbounis and Neiman 2014), this simple observation is contradictory with one of the Kaldor's facts.

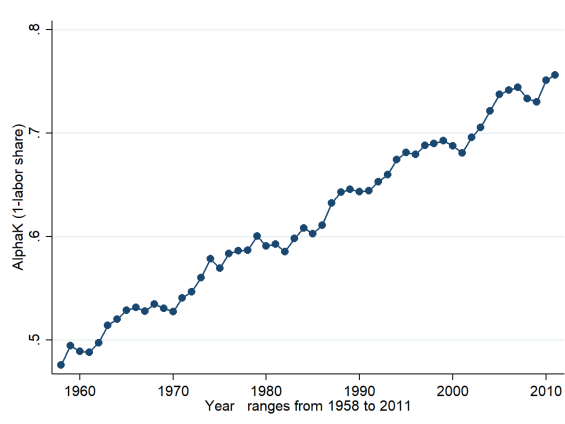


Figure 2: Aggregate Capital-Labor Intensity

Finding 2: There exists tremendous cross-industry and over-time dispersion in capital shares.

Ju, Lin and Wang (2015) documents that there is large cross-industry heterogeneity in capital intensities. We find as large heterogeneity in capital shares both cross-industry and over time. Figure 3 shows the distribution of capital share in 1958, 1980 and 2011 for US manufacturing industries. In 1958, the 90th percentile of capital share is 0.65, which is 1.86 times of the 10th percentile (0.35). While in 2011, the 90th percentile of capital share is 0.85, which is 1.55 times of the 10th percentile (0.55). From 1958 to 2011, the average capital share evolves from 0.47 to 0.69, with dispersion in capital share being persistent.

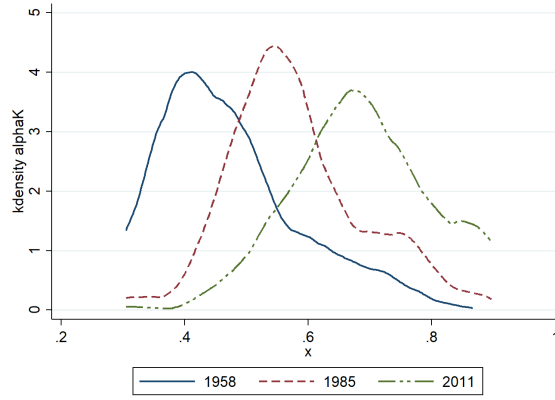


Figure 3: Distribution of Capital Intensity

Finding 3: The increasing capital share is largely contributed by within-industry change.

After observing the two patterns above, one would ask naturally, whether the increasing capital share is contributed by within-industry changes or changes in industry compositions. We resolve that question by running an industry-year level regression of capital shares as the following equation.

$$CapitalShare_{it} = \beta_0 + \beta_1 t + \delta_i + \epsilon_{it}$$

where i stands for industry and t stands for time. After controlling for sector fixed effect, the coefficient β_1 shows the within-industry changes in capital share. To be more rigorous, we also conduct the following regression of capital share using UNIDO dataset (covering 148 countries and 18 manufacturing sectors from 1963 to 2014), controlling for sector and country fixed effect.

$$CapitalShare_{cit} = \beta_0 + \beta_1 t + \delta_i + \gamma_c + \epsilon_{it}$$

where c stands for country, i stands for industry and t stands for time. After controlling for sector and country fixed effect, the coefficient β_1 indicates within industry changes in capital share. The regression results are shown in Table 1.

Column 1 shows the simple OLS regression using NBER-CES data. The coefficient on t shows the average effect of increasing capital share across industries. In column 2, we control for sector fixed effects. The coefficient on t represents the increasing capital share within industry. We observe a

Table 1: Within-industry Changes in Capital Shares

	(1)	(2)	(3)	(4)	(5)
	NBER_CES		UNIDO		
	cap_share	cap_share	cap_share	cap_share	cap_share
time	0.00401*** (85.03)	0.00404*** (175.2)	0.000215*** (4.63)	0.000510*** (11.3)	0.000782*** (19.47)
cons	0.475*** (319.06)	0.474*** (652.56)	0.587*** (410.18)	0.579*** (429.85)	0.569*** (62.31)
<i>Sector FE</i>	NO	YES	NO	Country	Country*Sector
<i>N</i>	24676	24676	74440	74440	74440
<i>r2</i>	0.227	0.559	0.000288	0.332	0.492

slightly larger coefficient of β_1 . It indicates that the increasing capital share is mainly contributed by within industry changes. Similarly, we use UNIDO dataset to conduct a country-industry-year regression of capital share w.r.t time. Column 3 shows the results for OLS regression, while in Column 4 we add country fixed effect, and in Column 5 we additionally controls for country by industry fixed effect. The increasing coefficient estimates of β_1 also indicates that within-industry changes of capital share mainly contribute to the aggregate changes.

Figure 4 is a more straightforward illustration of within-industry changes in capital shares. Each panel stands for one 2-digit ISIC industries in US. Scatter points in each window are capital shares of each 4-digit ISIC industries. We clearly observe increasing in capital share in each industry.

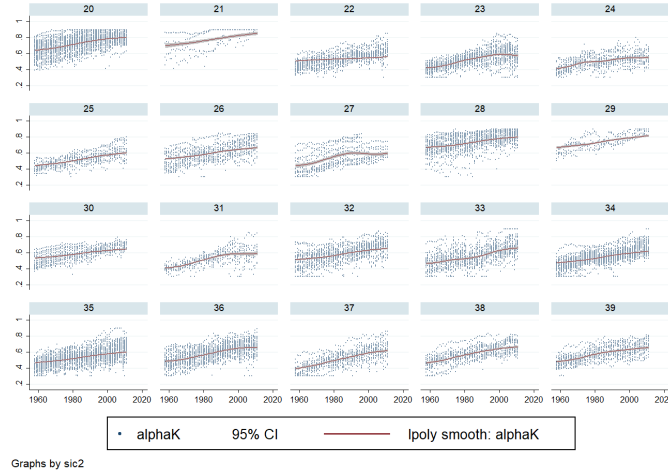


Figure 4: Capital Shares in 4-digit Industries

3 Model

3.1 Set-up

Consider a closed economy with a continuum of industries on $[0, 1]$ and a continuum of individuals on $[0, 1]$, for any $i \in [0, 1]$, in the industry i , there are a finite number of firms producing the good i , and all the individuals are homogeneous, each of which endows the initial physical capital $K_0 > 0$ and labor $L_0 > 0$, and denote $k_0 = K_0/L_0$.

We make some basic assumptions as follows.

A1. Each consumer is a utility maximizer, and the utility function of each consumer is

$$U((C_i)_{i \in [0,1]}) = \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho},$$

where $\rho \in [0, 1)$, $\theta_i > 0$ are constants, and θ_i as a function of $i \in [0, 1]$ is piece-wise smooth and satisfying $\int_0^1 \theta_i di = 1$, and $C_i \geq 0$ is the quantity of good i the individual consumes. If $\rho = 0$, then, the utility function is reduced to the Cobb-Douglas form:

$$U((C_i)_{i \in [0,1]}) = \int_0^1 \theta_i \ln C_i di,$$

A2. Each firm is a profit maximizer, and the possible production function of the industry i is

$$F_i(K, L) = ((a_i K)^{\rho_i} + (b_i L)^{\rho_i})^{1/\rho_i},$$

and

$$\left(\frac{a_i}{m_i} \right)^{\sigma_i} + \left(\frac{b_i}{n_i} \right)^{\sigma_i} = 1, \tag{1}$$

where $m_i > 0$, $n_i > 0$, $\rho_i < 1$, and $\sigma_i \neq 0$ are constants, satisfying $\sigma_i \rho_i > 0$. Our model generally allow different sectors vary in the elasticity of substitution between capital and labor in production function and technology frontiers. K and L are the physical capital and labor, respectively. And, a_i and b_i are the augmenting coefficients for capital and labor, respectively, which can be chosen according to (1), which, in turn, is the technology frontier of the industry i , similar with Caselli and Coleman (2006).

A3. For any $i \in [0, 1]$, it holds that

$$\rho < \frac{\sigma_i \rho_i}{\sigma_i - \rho_i} < 1,$$

A4. All the markets are competitive, including factor markets and goods markets.

A5. Every consumer owns a definite share of profit of every firm.

For convenience, for any $i \in [0, 1]$, denote

$$\varepsilon_i =: \frac{\sigma_i \rho_i}{\sigma_i - \rho_i}, \quad \tau_i =: \frac{\rho_i}{1 - \rho_i}, \quad \gamma_i =: \frac{m_i}{n_i}, \quad \delta_i =: \frac{\sigma_i \tau_i}{\sigma_i - \tau_i}.$$

Clearly,

$$\delta_i = \frac{\varepsilon_i}{1 - \varepsilon_i},$$

and **A3** is equivalent to that

$$\varepsilon_i > \rho, \quad \sigma_i > \tau_i, \quad \forall i.$$

Now, we give an interpretation for the meaning of ε_i . Clearly,

$$\frac{1}{\varepsilon_i} = \frac{1}{\rho_i} - \frac{1}{\sigma_i},$$

then, the more close between ρ_i and σ_i , the bigger ε_i is. Since

$$\frac{1}{1 - \rho_i}$$

is the elasticity of substitution between K_i and L_i , and, analogously, we can say that

$$\frac{1}{1 - \sigma_i}$$

is the elasticity of substitution between a_i and b_i , or, in mathematics, ρ_i determines in some sense the shape of the production function isoquant curve for industry i , and analogously, σ_i determines in some sense the shape of the technology frontier for industry i , then, ε_i captures joint features of the production function and the technology frontier of industry i , the bigger it is, the more similar the shape of the production function isoquant curve and that of the technology frontier of industry i are. For simplicity, we call ε_i the similitude degree in industry i which means roughly the degree of coordination of the choice of the augmenting coefficients for capital and labor (a_i and b_i) with the fundamental feature of the capital and labor in industry i (K_i and L_i). We will see below (Proposition 4) that the higher the similitude degree is, the more quickly the industry upgrades in technology (that is, the more capital intensive), along with the increase of capital per capita of the whole society.

Remark 1. The condition $\sigma_i \rho_i > 0$ means that σ_i and ρ_i have the same sign. That is, when the capital and labor are complimentary ($\rho_i < 0$, and therefore, the elasticity of substitution between capital and labor is $1/(1 - \rho_i) > 1$), then, the augmenting coefficients for capital and labor a_i and b_i are also complimentary; when the capital and labor are substitute ($\rho_i > 0$, and therefore, the elasticity of substitution between capital and labor is $1/(1 - \rho_i) < 1$), then the augmenting coefficients for capital and labor a_i and b_i are also substitute. Figure 5 shows an illustration of the technology frontier under different values of parameters. The left panel is an example when σ_i is larger than 0 and smaller than 1; the right panel is one when σ_i is negative. Note that when σ_i is larger than 1, the technology frontier is similar to the left panel, but concave in shape.

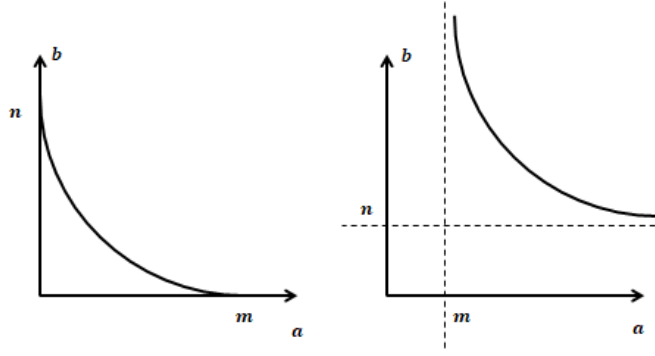


Figure 5: Illustration of Technology Frontiers

The condition $\rho < \varepsilon_i$ means that comparing with each ε_i , ρ is relatively small, or equivalently, the elasticity of substitution between different goods $1/(1 - \rho)$ is relatively small.

Concerning the firm's profit maximization problem, by solving the embedded cost minimization problem, one can easily find that under the condition $\varepsilon_i < 1$, the optimal choice of a_i, b_i is an interior solution, under the assumption **A3**, and it will be corner solution, under the assumption **A3'** in section 3.7.

In our following discussion, without any loss of generality, we may make a further assumption.

A6. In each industry, there is only one firm.

We give an explanation for this assumption. First of all, we can prove (see the proof of theorem 1 in Appendix I) that in each industry, facing any given price system, any two firms will take the same technology and the same capital-labor ratio, and hence, since the production function is first order homogeneous, then, we can combine all of these firms to one firm, which produce the sum of the productions of these firms and use up the sum of the capitals and labors used in the production of these firms. Therefore, we can make the assumption **A6**.

In the end of this section, we mention that in each sector, the total possibility production set is the union of the production possibility sets for each possible technology, any one of which is convex, but their union is not. And hence, the classical Arrow-Debreu sufficiency conditions for the existence of Walrasian equilibrium are not satisfied completely, but we will see that in our setting, the equilibrium exists and is unique.

Another problem is worth mentioning. That is, what about the case, where multiple technologies are allowed to take simultaneously for any one firm. However, this problem is equivalent to the above problem in some sense. In fact, if a firm takes two technology simultaneously, say, technology 1 and technology 2, then, according to the above analysis, this is equivalent to the case, where there are two firms, firm 1 takes the technology 1, firm 2 takes the technology 2, and therefore, facing the same price system, there exist two firms, which take different technologies, and this would contradict to the above analysis.

In one word, the assumption **A6** is acceptable.

3.2 General equilibrium

Under the condition **A3**, all the firms in the same industry have the same interior solution for the technology choice, and noticing that the production functions are all first order homogeneous, and hence, for simplicity, we make a further assumption:

A6. In each industry, there is only one firm.

Now, we give our formal definition of general equilibrium.

Definition 1. $(C_i^*, a_i^*, b_i^*, K_i^*, L_i^*; p_i^*, r^*, \omega^*)_{i \in [0,1]}$ is an equilibrium, if

(I)

$$(C_i^*)_{i \in [0,1]} \in \arg \max_{(C_i)_{i \in [0,1]}} U((C_i)_{i \in [0,1]}),$$

s.t.

$$\int_0^1 p_i^* C_i^* di \leq r^* K_0 + \omega^* L_0;$$

(II) for any $i \in [0, 1]$,

$$(a_i^*, b_i^*, K_i^*, L_i^*) \in \arg \max_{a, b, K, L} \left\{ p_i^* ((aK)^{\rho_i} + (bL)^{\rho_i})^{1/\rho_i} - r^* K - \omega^* L \right\},$$

s.t.

$$\left(\frac{a}{m_i} \right)^{\sigma_i} + \left(\frac{b}{n_i} \right)^{\sigma_i} = 1;$$

(III)

$$\begin{aligned} C_i^* &= ((a_i^* K_i^*)^{\rho_i} + (b_i^* L_i^*)^{\rho_i})^{1/\rho_i}, \quad i \in [0, 1], \\ \int_0^1 K_i^* di &= K_0, \\ \int_0^1 L_i^* di &= L_0. \end{aligned}$$

Remark 2. One can see that if $(C_i^*, a_i^*, b_i^*, K_i^*, L_i^*; p_i^*, r^*, \omega^*)_{i \in [0,1]}$ is an equilibrium, then,

$$r^* > 0, \quad \omega^* > 0,$$

and

$$C_i^* > 0, \quad p_i^* > 0, \quad \text{a.s.}$$

where "a.s." stands "almost surely with respect to $i \in [0, 1]$ " in Lebesgue measure. For simplicity, we omit the notation "a.s." in the sequel, when it is required.

Remark 3. It's easy to see that in equilibrium, each firm's maximized profit must be 0.

3.3 Main results

Now, we state our main results.

Theorem 1. For any $k_0 > 0$, there exists a unique equilibrium $(C_i; a_i, b_i, K_i, L_i; p_i, r, \omega)_{i \in [0,1]}$, which is determined by the following equations: for any $i \in [0, 1]$,

$$\begin{aligned} C_i &= ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i}, \\ K_i &= \frac{(\theta_i m_i^\rho (1 + z_i^{-1})^{\rho/\varepsilon_i - 1})^{1/(1-\rho)}}{\int_0^1 (\theta_j m_j^\rho (1 + z_j^{-1})^{\rho/\varepsilon_j - 1})^{1/(1-\rho)} dj} K_0, \\ L_i &= \frac{(\theta_i n_i^\rho (1 + z_i)^{\rho/\varepsilon_i - 1})^{1/(1-\rho)}}{\int_0^1 (\theta_j n_j^\rho (1 + z_j)^{\rho/\varepsilon_j - 1})^{1/(1-\rho)} dj} L_0, \\ a_i &= m_i (1 + z_i^{-1})^{-1/\sigma_i}, \\ b_i &= n_i (1 + z_i)^{-1/\sigma_i}, \\ \frac{\omega}{p_i} &= n_i (1 + z_i)^{1/\delta_i}, \\ \frac{\omega}{r} &= z, \end{aligned}$$

where

$$z_i = (\gamma_i z)^{\delta_i},$$

and z is determined by

$$k_0 = \frac{z^{1/(1-\rho)} \int_0^1 [\theta_i m_i^\rho (1 + (\gamma_i z)^{-\delta_i})^{\rho/\varepsilon_i - 1}]^{1/(1-\rho)} di}{\int_0^1 [\theta_j n_j^\rho (1 + (\gamma_j z)^{\delta_j})^{\rho/\varepsilon_j - 1}]^{1/(1-\rho)} dj}. \quad (2)$$

And for this equilibrium, for any $i \in [0, 1]$,

$$\begin{aligned} \left(\frac{a_i}{r_i}\right)^{\tau_i} &= \left(\frac{m_i}{r_i}\right)^{\delta_i} = \left(\frac{a_i}{m_i}\right)^{\sigma_i} = \frac{z_i}{1 + z_i} = \alpha_i, \\ \left(\frac{b_i}{\omega_i}\right)^{\tau_i} &= \left(\frac{n_i}{\omega_i}\right)^{\delta_i} = \left(\frac{b_i}{n_i}\right)^{\sigma_i} = \frac{1}{1 + z_i} = \beta_i, \\ z_i &= (\eta_i/\gamma_i)^{\sigma_i} = (\eta_i z)^{\tau_i} = (\eta_i k_i)^{\rho_i} = (\gamma_i k_i)^{\varepsilon_i} = k_i/z, \end{aligned}$$

where

$$r_i = \frac{r}{p_i}, \quad \omega_i = \frac{\omega}{p_i}, \quad k_i = \frac{K_i}{L_i}, \quad \eta_i = \frac{a_i}{b_i}, \quad \alpha_i = \frac{r_i K_i}{C_i}, \quad \beta_i = \frac{\omega_i L_i}{C_i}.$$

Here, k_i is the capital per capita in the industry i , η_i can be considered as a relative augmenting coefficient of capital to labor in the industry i , α_i and β_i are the capital share and labor share in the industry i , and z is well-defined (See Lemma 1 in the Appendix).

From this theorem, we get our first proposition.

Proposition 1.(Endowment Driven Equilibrium) The choice of technologies is determined by the structure of factor endowments and the taste for commodities of the people. In particular, for different economies, if the tastes for commodities of the people are considered the same, and all the technologies are common knowledge for the whole humanity, then, the choice of technologies for each economy is determined only by the structure of its factor endowments.

The observed capital intensity, either defined as capital-labor ratio or capital income share, are endogenous in our model. The two measures are coherent.

Corollary 1. In equilibrium, for any $i \neq j \in [0, 1]$,

$$\alpha_i > \alpha_j \iff k_i > k_j.$$

Concerning the η_i , the problem is more complicated. In general, we can not conclude that $\eta_j > \eta_i$ means that the industry j is more capital intensive than the industry i .

Corollary 2. In equilibrium, for any $i \neq j \in [0, 1]$,

(i) if one of the following two conditions holds:

$$\sigma_i = \sigma_j > 0, \quad \gamma_i = \gamma_j, \tag{3}$$

or

$$\rho_i = \rho_j > 0, \tag{4}$$

then,

$$\alpha_i > \alpha_j \iff \eta_i > \eta_j;$$

(ii) if one of the following two conditions holds:

$$\sigma_i = \sigma_j < 0, \quad \gamma_i = \gamma_j, \tag{5}$$

or

$$\rho_i = \rho_j < 0, \tag{6}$$

then,

$$\alpha_i > \alpha_j \iff \eta_i < \eta_j.$$

Conditions (3) and (5) mean that the technology frontiers for the industry i and the industry j are of the same type, having the same shape, the only difference between them is in their "sizes", or, put it another way, roughly, one frontier is an enlarged version of another, enlarged in both directions

in the same magnitude. If (3) or (5) holds, then we say that industry i and industry j have similar technology frontier.

Conditions (4) and (6) mean that the elasticities of substitution between capital and labor in the industry i and industry j are equal.

Remark 4. Corollary 1 tells us that α and k are coherent. Corollary 2 says that for industries with similar technology frontiers or the same elasticity of substitution between capital and labor, if the capital and labor are substitute in those industries, then, α and η are coherent as well; if the capital and labor are complimentary in those industries, then, the relationship between α and η is negative.

We are also interested in the relationship between η_i and γ_i . We wonder whether it holds $\eta_i > \eta_j$ or not, in the case, where $\gamma_i > \gamma_j$.

In general, it does not hold. From Theorem 1, we have that for any i ,

$$\eta_i^{\sigma_i - \tau_i} = z^{\tau_i} \gamma_i^{\sigma_i}.$$

It follows the following corollary.

Corollary 3. In equilibrium, for two industries i and j with $\sigma_i = \sigma_j$, $\rho_i = \rho_j$, then, if $\sigma_i = \sigma_j > 0$, then,

$$\eta_i > \eta_j \iff \gamma_i > \gamma_j;$$

if $\sigma_i = \sigma_j < 0$, then,

$$\eta_i > \eta_j \iff \gamma_i < \gamma_j.$$

This result shows us how to interpret the economic meanings of the parameters η_i and γ_i , and how to "define" the relative more capital intensive industries in our setting.

3.4 Social Planner's Problem

The social welfare can be measured by the individual's utility function value in this case. Therefore, the social planner's problem is as follows.

$$\begin{aligned} \max \quad & \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho}, \\ \text{s.t.} \quad & C_i = ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i}, \quad i \in [0, 1] \\ & (a_i/m_i)^{\sigma_i} + (b_i/n_i)^{\sigma_i} = 1, \quad i \in [0, 1], \\ & \int_0^1 K_i di = K_0, \quad \int_0^1 L_i di = L_0. \end{aligned}$$

One can prove that this problem has unique solution. The proof is standard, we omit it.

According to the first theorem of welfare economics, the outcome of the general equilibrium must be the solution of the social planner's problem, that is, the outcome of the general equilibrium is Pareto optimal. (One can verify this statement by solving the social planner's problem directly.)

The corresponding maximized social welfare value³ is

$$W = \left(L_0 \frac{\int_0^1 \theta_i^{1/(1-\rho)} (n_i(1+z_i)^{1/\delta_i})^{\rho/(1-\rho)} di}{\int_0^1 [\theta_j n_j^\rho (1+z_j)^{\rho/\varepsilon_j-1}]^{1/(1-\rho)} dj} \right)^{1/\rho}.$$

By the envelope theorem, one can get that W is strictly increasing with respect to k_0 , n_i and m_i , for any $i \in [0, 1]$.

That is, if the capital endowment grows or some technology progress takes place, then, the society is made better off.

Remark 5. We have seen that for any industry, the appropriate technology, (a_i, b_i) , is determined essentially by the proportion z_i , which depends on z , which, in turn, is determined by the initial factor endowments (K_0, L_0) . If we choose another z , not adapted to the initial factor endowments, and this would give incorrect z_i , and finally, incorrect (a_i, b_i) , and this would induce a loss of welfare, i.e. factor market distortions bring welfare loss.

3.5 Comparative Static Analysis

From the above closed form solution of the equilibrium, it follows a corollary directly.

Corollary 4. For any $i \in [0, 1]$, with respect to k_0 , α_i , k_i , ω_i are strictly increasing; β_i , r_i is strictly decreasing; ω/r and W are strictly increasing; if $\rho_i > 0$, then, a_i is strictly increasing and b_i is strictly decreasing; and if $\rho_i < 0$, then, a_i is strictly decreasing and b_i is strictly increasing.

Put into words, we get the following proposition.

Proposition 2. (Endowments and Technology Choices) Along with the increase of capital per capita of the society, each industry will experience a technology upgrading from labor-intensive to capital-intensive, and in this process, the relative price of capital to labor is decreasing; and the labor share in each industry is decreasing; the capital per capita in each industry is increasing.

3.6 Industrial Structure

For any $i \in [0, 1]$, denote the monetary total output of the industry i as

$$M_i = p_i C_i,$$

and hence, the total monetary output of the whole economy, the GDP, is

$$M = \int_0^1 M_i di.$$

Clearly, by the definition of equilibrium, we have that $M = I$, where I is just the total income of the individual.

³Under the additional assumption that $m_i > 0$, $n_i > 0$, ρ_i and σ_i , as functions of $i \in [0, 1]$, are all piece-wise smooth.

For any $i \in [0, 1]$, let

$$w_i = M_i/M,$$

which is the proportion of the industry i in the whole economy.

The distribution $\{w_i\}_{i \in [0, 1]}$ can be used to express the industrial structure of this economy. It relates to the distribution of $\{k_i\}_{i \in [0, 1]}$.

We now investigate the change of these two distributions along with the change of k_0 .

By the proof the Theorem 1 in the Appendix, we know that for any $i \in [0, 1]$,

$$w_i = \frac{\left(\theta_i^{1/\rho} n_i (1 + z_i)^{1/\delta_i}\right)^{\rho/(1-\rho)}}{\int_0^1 \left(\theta_j^{1/\rho} n_j (1 + z_j)^{1/\delta_j}\right)^{\rho/(1-\rho)} dj}.$$

For any $i \neq j$, it's easy to varify that

$$\frac{\partial}{\partial k_0} \left(\frac{w_i}{w_j} \right)$$

has the same sign with $k_i - k_j$.

And hence, we get a result concerning the change of the distribution of $\{w_i\}_{i \in [0, 1]}$. We write it as a proposition.

Proposition 3. (Endowment and Industrial Structure) Along with the increase of the capital per capita of the society, the industrial structure changes accordingly, it tends more and more towards capital-intensive industries.

Now, we look at the change of the distribution of $\{k_i\}_{i \in [0, 1]}$ along with the increase of k_0 .

Noticing that for any $i \neq j$,

$$\frac{k_i}{k_j} = \frac{\gamma_i^{\delta_i}}{\gamma_j^{\delta_j}} z^{\delta_i - \delta_j},$$

we get that along with the increase of k_0 , k_i increases more quickly (slowly) than k_j , if and only if $\delta_i > (<) \delta_j$, which is equivalent to $\varepsilon_i > (<) \varepsilon_j$.

And hence, we get a proposition as follows.

Proposition 4. (Endowments and Capital Intensities) Along with the increase of the capital per capita of the society, among all the sectors, the higher the similitude degree is, the more quickly its capital per capita increases. In particular, for any two sectors with the same similitude degrees, the ratio of their capitals per capita will remain constant.

Corollary 5. If $\varepsilon_i = \varepsilon$, for all $i \in [0, 1]$, where $\varepsilon \in (\rho, 1)$ is some constant, then, for any i, j ,

$$\frac{k_i}{k_j} \equiv \left(\frac{\gamma_i}{\gamma_j} \right)^\delta,$$

where

$$\delta = \frac{\varepsilon}{1 - \varepsilon},$$

and

$$\frac{\partial}{\partial k_0} \left(\frac{w_i}{w_j} \right) > (=, <) 0,$$

if and only if

$$\gamma_i > (=, <) \gamma_j.$$

Therefore, in the case, where the parameter $\varepsilon_i = 1/(\frac{1}{\rho_i} - \frac{1}{\sigma_i})$ for all sectors are the same, their capitals per capita will increase in the same pace, and hence, "the capital intensiveness structure" of the economy will remain still, that is, a relative capital intensive industry will always remain relative capital intensive, a relative labor intensive industry will always remain relative labor intensive, (noticing that the real capital intensiveness for any industry will increase along with the increase of k_0), or, in other words, the relative location of any industry will remain the still in the economy, and the monetary output of a relative capital intensive industry will increase quickly than a relative labor intensive industry so that the economy will more and more tend to tilt to capital intensive industries.

For example, suppose that $\varepsilon_i = \varepsilon$, for all $i \in [0, 1]$, and γ_i is strictly increasing with respect to i , then, on the interval $[0, 1]$, among all the industries, from left to right, the industry is more and more capital intensive, and this situation will remain still, along with the increase of k_0 , and the distribution of $\{w_i\}_{i \in [0, 1]}$ will tend to put more and more weight on the relative right place, that is, the economy will tend to more and more tilt to capital intensive industries.

3.7 Corner Solution

In the above setting, an interior solution occurs, in which the assumption $\sigma_i > \tau_i$ is essential.

In this section, for simplicity and for comparison, we consider the discrete case, where the economy has only n sectors, and $\rho_i > 0$, $\sigma_i > 0$ for all $i = 1, \dots, n$. And even more, replacing **A3**, we make an assumption:

A3'. For any $i = 1, \dots, n$, it holds that

$$\varepsilon_i > \rho, \quad \sigma_i < \tau_i.$$

In this case, by the similar method used in the proof of Theorem 1 in Appendix 1, we see that each firm has a corner solution in its technology choice problem, that is, for a firm in the industry i , the optimal choice of a_i, b_i is

$$a_i = m_i, \quad b_i = 0,$$

and accordingly,

$$K_i > 0, \quad L_i = 0;$$

or

$$a_i = 0, \quad b_i = n_i,$$

and accordingly,

$$K_i = 0, \quad L_i > 0;$$

or both. In the last case, where both of the corners are all optimal, then, in this industry, different firms may choose different technologies, but, of course, only one of the corners.

And hence, holding **A3'**, the assumption **A6** we made above is not suitable, because under this assumption, the general equilibrium may not exist. Therefore, under **A3'**, replacing **A6**, we make an assumption that in each industry, there are two firms, which take different corner technologies.

And, because the production functions are all first order homogeneous, an equivalent assumption is as follows:

A6'. There is only one firm in each industry, but it can take the two corner technologies simultaneously to produce its products.

Under this assumption, we have that for the firm in the industry i , if the capital and labor it demands are K_i, L_i respectively, then, the output it produces is

$$Y_i = m_i K_i + n_i L_i,$$

and if $K_i > 0$, then, it must take the technology

$$a_i = m_i, \quad b_i = 0.$$

if $L_i > 0$, then, it must take the technology

$$a_i = 0, \quad b_i = n_i.$$

The concept of general equilibrium can be defined similarly. And hence, to express the equilibrium, without any loss, we omit to write out the a_i, b_i and C_i explicitly.

For simplicity, we make a further assumption that $\rho = 0$, and hence, the individual utility function is of Cobb-Douglas form.

We say that industry j is more capital intensive potentially (or higher) than industry i , if $\gamma_j > \gamma_i$.

Now, we rank $\{\gamma_1, \dots, \gamma_n\}$ in an increasing order. For simplicity, we assume that

$$\gamma_1 \leq \gamma_2 \leq \dots \leq \gamma_n. \quad (7)$$

That is, throughout the industry 1 to the industry n , the potential capital intensities are increasing.

For any $i = 1, \dots, n-1$, denote

$$A_t = \frac{1}{\gamma_t} \frac{\sum_{j>t} \theta_j}{\sum_{j \leq t} \theta_j},$$

$$A'_t = \frac{1}{\gamma_{t+1}} \frac{\sum_{j>t} \theta_j}{\sum_{j \leq t} \theta_j},$$

and put

$$A'_0 =: \infty, \quad A_n =: 0.$$

We have that

$$\infty = A'_0 > A_1 \geq A'_1 > \dots > A_{n-1} \geq A'_{n-1} > A_n = 0.$$

Now, we state our result.

Theorem 2. The equilibrium exists. And $(K_i, L_i; p_i, r, \omega)_{i=1, \dots, n}$ is an equilibrium, if and only if it satisfies that

(I) for any $i = 1, \dots, n$,

$$I_i = \theta_i I_0,$$

where

$$I_i = rK_i + \omega L_i, \quad \forall i = 0, 1, \dots, n;$$

(II) for any $i = 1, \dots, n$, if

$$\gamma_i \geq \frac{r}{\omega},$$

then,

$$L_i = 0, \quad p_i m_i = r;$$

if

$$\gamma_i \leq \frac{r}{\omega},$$

then,

$$K_i = 0, \quad p_i n_i = \omega;$$

(III) if

$$A_t \geq k_0 \geq A'_t, \quad t = 1, \dots, n-1,$$

then,

$$\frac{r}{\omega} = \frac{1}{k_0} \frac{\sum_{j>t} \theta_j}{\sum_{j \leq t} \theta_j};$$

if

$$A'_{t-1} > k_0 > A_t, \quad t = 1, \dots, n,$$

then,

$$\frac{r}{\omega} = \gamma_t;$$

(IV)

$$\sum_{i=1}^n K_i = K_0, \quad \sum_{i=1}^n L_i = L_0.$$

Let's look at a simple special case.

Corollary 6. Suppose that every inequality in (7) is strict inequality. Then, the equilibrium $(K_i, L_i; p_i, r, \omega)_{i=1, \dots, n}$ is unique, which is determined as follows:

(I) if

$$A_t \geq k_0 \geq A'_t, \quad t = 1, \dots, n-1,$$

then, for any $i \leq t$,

$$K_i = 0, \quad L_i = \frac{\theta_i}{\sum_{j \leq t} \theta_j} L_0, \quad p_i n_i = \omega,$$

and for any $i > t$,

$$L_i = 0, \quad K_i = \frac{\theta_i}{\sum_{j>t} \theta_j} K_0, \quad p_i m_i = r,$$

and and

$$\frac{r}{\omega} = \frac{1}{k_0} \frac{\sum_{j>t} \theta_j}{\sum_{j\leq t} \theta_j};$$

(II) if

$$A'_{t-1} > k_0 > A_t, \quad t = 1, \dots, n,$$

then, for any $i < t$,

$$K_i = 0, \quad L_i = \theta_i (\gamma_t K_0 + L_0), \quad p_i n_i = \omega,$$

for any $i > t$,

$$L_i = 0, \quad K_i = \theta_i (K_0 + L_0/\gamma_t), \quad p_i m_i = r,$$

and

$$\begin{aligned} K_t &= \sum_{j\leq t} \theta_j K_0 - \frac{1}{\gamma_t} \sum_{j>t} \theta_j L_0, \\ L_t &= \sum_{j\geq t} \theta_j L_0 - \gamma_t \sum_{j<t} \theta_j K_0, \\ p_t m_t &= r, \quad p_t n_t = \omega, \end{aligned}$$

and

$$\frac{r}{\omega} = \gamma_t.$$

Summing up the above analysis, in this setting, we get the following proposition.

Proposition 5. (Technology upgrading) Along with the increasing of the capital per capita of the society, all the industries will take technology upgrading sequentially, each industry (except for the first industry and the last industry) will experience three phases of technology upgrading: pure labor technology, mixed technology, and pure capital technology, the first industry experiences only two phases: pure labor technology, and mixed technology; the last industry experiences only two phases: mixed technology, and pure capital technology. And at any level of the capital per capita, there is only one industry taking the mixed technology, all lower industries take pure labor technology, all higher industries take pure capital technology. Along with the increasing of the capital per capita from 0 to ∞ , the order of technical upgrading is from higher industries to lower industries one by one sequentially.

3.8 Lontief case

If in the basic setting in 3.1, holding all the assumptions, and let $\rho_i = -\infty$ for some (may or may not be all) $i \in [0, 1]$, then, the technologies in these industries will be reduced to Lontief type, that is, for these industries, the production functions will be

$$Y_i = \min\{a_i K_i, b_i L_i\}.$$

And, in this case, for these industries, we have

$$\varepsilon_i = -\sigma_i, \quad \tau_i = -1, \quad \delta_i = \frac{\sigma_i}{1 + \sigma_i},$$

and, assumption A3 will be reduced to

$$\rho < -\sigma_i < 1.$$

In this case, all of our main results will remain true. That is, the Lontief case is a special case of our setting. More precisely, even if the production functions in some of the industries are changed from the ordinary CES function to a Lontief function, the equilibrium will also exist and is unique, and can be determined by the same method as in Theorem 1.

In the end of this subsection, we discuss the Lontief case in another setting of technology choice. Let's first consider a one-sector model, the production function is of Lontief type:

$$Y = \min\{aK, bL\},$$

where Y is the consumption good, K, L are capital and labor respectively, $a > 0, b > 0$ are technology parameters, the set of all possible (a, b) is denoted as T , which is just the technology set. Suppose there is only one individual, owning the initial endowments K_0, L_0 .

If T is only a single-point set, then, obviously, the Walrasian equilibrium exists always. We denote the prices of capital and labor as r and ω respectively, and normalize the price of the consumption good as 1.

If

$$\frac{b}{a} = \frac{K_0}{L_0},$$

then, the equilibria are multiple, and (r, ω) is the equilibrium prices, if and only if it satisfies

$$1 = \frac{r}{a} + \frac{\omega}{b}.$$

If

$$\frac{b}{a} > \frac{K_0}{L_0},$$

then, the unique Walrasian equilibrium prices are $r = a, \omega = 0$. In this case, there exists free disposal of labor. That is, with respect to this technology, the social labor supply is excessive. The free disposal equilibrium labor demand is any $L \in [K_0 a/b, L_0]$.

If

$$\frac{b}{a} < \frac{K_0}{L_0},$$

then, the unique Walrasian equilibrium prices are $r = 0, \omega = b$. In this case, there exists free disposal of capital. That is, with respect to this technology, the social capital supply is excessive. The free disposal equilibrium capital demand is any $K \in [L_0 b/a, K_0]$.

If the technology set T is a two-point set, e.g.

$$T = \{(a_1, b_1), (a_2, b_2)\},$$

and suppose that

$$\frac{b_1}{a_1} < \frac{b_2}{a_2},$$

then, if only one technology can be taken, then, the Walrasian equilibrium without free disposal exists, if and only if there is a $i \in \{1, 2\}$ such that

$$\frac{K_0}{L_0} = \frac{b_i}{a_i}.$$

If the two technologies can be chosen simultaneously, then, the Walrasian equilibrium without free disposal exists, if and only if

$$\frac{b_1}{a_1} \leq \frac{K_0}{L_0} \leq \frac{b_2}{a_2}.$$

Otherwise, there only exists Walrasian free disposal equilibrium.

One can extend this discussion to multiple-sector case, and get similar results. As an example, we here discuss a two-sector model, and we only discuss the pure technology case, that is, any firm is only allowed to take one technology. And, for simplicity, we assume that any one of the two sectors, there is only one firm, and in this economy, there is only one individual with endowments K_0, L_0 .

Suppose the utility function of the individual is

$$U(C_1, C_2) = C_1^{\theta_1} C_2^{\theta_2},$$

where $\theta_1 > 0$, $\theta_2 > 0$ and $\theta_1 + \theta_2 = 1$. And suppose that for any $i \in \{1, 2\}$, in the industry i , the production function is

$$Y_i = \min\{a_i K_i, b_i L_i\},$$

and the technology set is T_i .

Then, one can prove that the Walrasian equilibrium without free disposal exists, if and only if there exist $(a_1, b_1) \in T_1$, $(a_2, b_2) \in T_2$ such that K_0/L_0 is located between

$$\theta_1 \frac{b_1}{a_1} + \theta_2 \frac{b_2}{a_2} \quad \text{and} \quad \left(\theta_1 \frac{a_1}{b_1} + \theta_2 \frac{a_2}{b_2} \right)^{-1}.$$

Otherwise, there exist only Walrasian free disposal equilibria.

After all, in the Lontief case, if free disposal is allowed, then, the equilibrium exists always. If free disposal is not allowed, then, the equilibrium may or may not exist, which depends, partly, on the structure of the technology set: if the technology set is too poor, then, the equilibrium is not likely to exist; if it is quite rich, then, the equilibrium is likely to exist.

This is a difference between the ordinary CES case in 3.1 (including the Cobb-Douglas case) and the Lontief case here. In the setting in 3.1, the equilibrium without free disposal and the equilibrium with free disposal are equivalent.

In section 7, we will discuss the equilibrium existence problem in another setting.

4 Empirical Patterns of Technology Choices

In this section, we first compute the technology choices and technology frontiers from observed data of endowment and factor returns. Further we show empirical patterns consistent with our model predictions.

4.1 Data description

We compute technology choices for the US manufacturing industries. We use the NBER-CES Manufacturing Industry Data for the US, which covers 459 ISIC within the manufacturing sectors from 1958 to 2011. The key variables we use include the value added y , real capital stock k , total employment l as well as wage payment, of each industry in each year. We divide the wage payment by employment for each industry to obtain wage. The rental price is computed as $(y - wl)/k$. The value added is normalized by price index of shipment. The capital stock is provided in real terms. The wage is deflated by annual CPI, provided by the Penn World Table 9.0. The data we use are summarized in the following Table 2. The total observation is 24,674 in 54 years.

Variable	Obs	Mean	Std. Dev.	Min	Max
emp	24676	36.3438	51.1565	0.1	565.4
pay	24676	784.571	1479.516	5	22245.3
cap	24676	2436.416	5844.753	3.7	105477.7
vadd	24676	2382.981	5506.654	10.2	111665.7

4.2 Computation of Technology Choices

Now, we compute the technology choice of a_i, b_i . By Corollary 1, we know that for any i ,

$$a_i = \frac{r}{p_i} \left(\frac{rK_i}{rK_i + wL_i} \right)^{\frac{1-\rho_i}{\rho_i}},$$

$$b_i = \frac{w}{p_i} \left(\frac{wL_i}{rK_i + wL_i} \right)^{\frac{1-\rho_i}{\rho_i}}.$$

And hence, we are able to solve for a_i and b_i using data on K_i, L_i, r and w , after calibrating parameter ρ_i .

We assume $\rho_i = \rho_0$ for all i , where ρ_0 is some constant. The parameter ρ_0 is determined by the elasticity of substitution between effective capital and labor $1/(1 - \rho_0)$, which has attracted a considerable amount of attention in macro literature. Leon-Ledesma et.al (2010) provides a summary of capital-labor substitution elasticity in production for US. Most practices in literature suggest the value to be above 0.5 and lower than 1, suggesting capital and labor in production function to be complements, instead of substitutes. For example, Bodkin and Klein (1967) suggests 0.5 to 0.7, Panik (1976) suggests 0.76, Leon-Ledesma and Satchi (2017) suggests 0.2, etc. Wang et.al (2018) and Knoblach et.al (2016) provide a nice review. Knoblach et.al (2016) utilizes 738 estimates from

41 studies published between 1961 and 2016 and finds the estimates of long-run elasticity lies in the range between 0.6 to 0.7. In our exercise, we try different values of $1/(1 - \rho_0)$ between 0.5 and 0.8.

4.3 Estimating Technology Frontiers

After computing technology choices a_i and b_i , we follow Caselli and Coleman (2006) to estimate each industry's technology frontiers $(\frac{a_i}{m_i})^{\sigma_i} + (\frac{b_i}{n_i})^{\sigma_i} = 1$ in each year.

We assume $\sigma_i = \sigma$ for all i , where σ is some constant. By Corollary 1, we get that for any i ,

$$\eta_i^{\sigma - \rho_0} = k_i^{\rho_0} \gamma_i^{-\sigma},$$

it follows that

$$\log \eta_i = \frac{\rho_0}{\sigma - \rho_0} \log k_i - \frac{\sigma}{\sigma - \rho_0} \log \gamma_i.$$

Then, we conduct the following regression:

$$\log \eta_{it} = \beta \log k_{it} + \epsilon_{it},$$

where β is the regression coefficient, and ϵ_{it} is the error.

From this regression, we can get the estimates of σ and γ_i . Together with the formula of technology frontier, we have estimates of m_i and n_i for each industry in each year. The following Table 3 summarizes our estimated of parameters. We can see the values are consistent with technical assumption A3.

Table 3: Table of Coefficients				
Parameter	σ	τ	ρ_0	ε
Value	-0.30	-0.50	-1.00	0.43

4.4 Empirical Findings

Finding 1: The change in technology choices with increasing capital endowment.

Firstly, we test Proposition 2 of the model. Table 4 shows results of a regression where the dependent variables is technology choice η_{it} (in log), and independent variables are aggregate endowment K_0/L_0 (in log) and the technology frontiers we computed from section 4.3 m_{it}/n_{it} (in log). We control for industry fixed effect in each regression, so the coefficients captures within-industry response of technology choices to the increase in aggregate capital endowment and changes in technology frontiers.

Column 1 uses the benchmark calibration of coefficient ρ_0 . We find the results are robust to different values of calibrated coefficient ρ_0 . We find the coefficient on capital endowment and on technology frontiers m_{it}/n_{it} are both negative, consist with the model. With more abundant capital endowment, US industries choose more labor-efficient technologies.

Figure 6 is a more straightforward illustration of the result. The graph shows the local polynomial fitness curve of the technology choices a and b for all US manufacturing industries along time.

Table 4: Technology Choices, Endowments and Frontiers

	(1)	(2)	(3)	(4)
	$\log(\eta)$	$\log(\eta)$	$\log(\eta)$	$\log(\eta)$
$1/(1 - \rho_0)$	0.4	0.5	0.6	0.7
$\log(K_0/L_0)$	-	-	-	-
	0.485*** (-17.76)	0.554*** (-19.77)	0.656*** (-22.49)	0.826*** (-26.23)
$\log(m/n)$	-	-	-0.024*	0.017
	0.066*** (-6.24)	0.049*** (-4.07)	(-1.68)	-0.92
N	25386	25386	25386	25386
r^2	0.313	0.317	0.349	0.424

Consistent with our regression findings, we observe rising in labor augmenting factor b and lowering capital augmenting factor a .

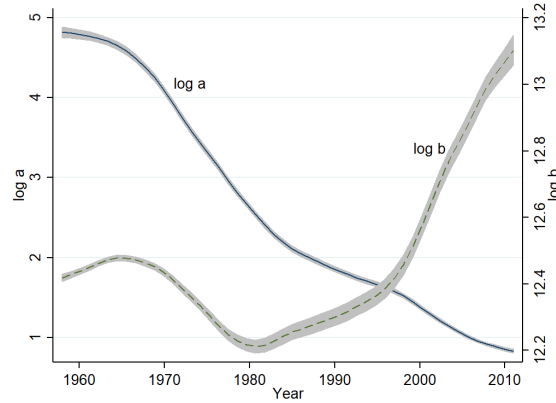


Figure 6: Technology Choices in US

Finding 2: Structural Change with increasing capital endowment.

Next, we test the Model Proposition 3 by investigating patterns of structural change, regarding capital intensity. The fact is first documented by Ju et.al (2015), where industries are defined with its capital intensity. We keep the definition of industry of SIC, and look into structural change. Figure 7 shows the distribution of industrial sizes over capital intensity (capital income share) in the US in different years. In the left panel, we use the 4-digit industries. The horizontal axis is the sectoral average capital share, measured by one minus labor income share in value added. The vertical axis is the sectoral value added share in total value added of manufacturing industries. The line is local polynomial fit curve. The graph shows that, industries with high capital share gain larger share in total economy; while the labor intensive industries lose share. In the right panel, we show the results when industries are at 2-digit. The results are qualitatively the same. To sum up, the pattern confirms structural change towards capital intensive industries, with more rigorous definition of industries.

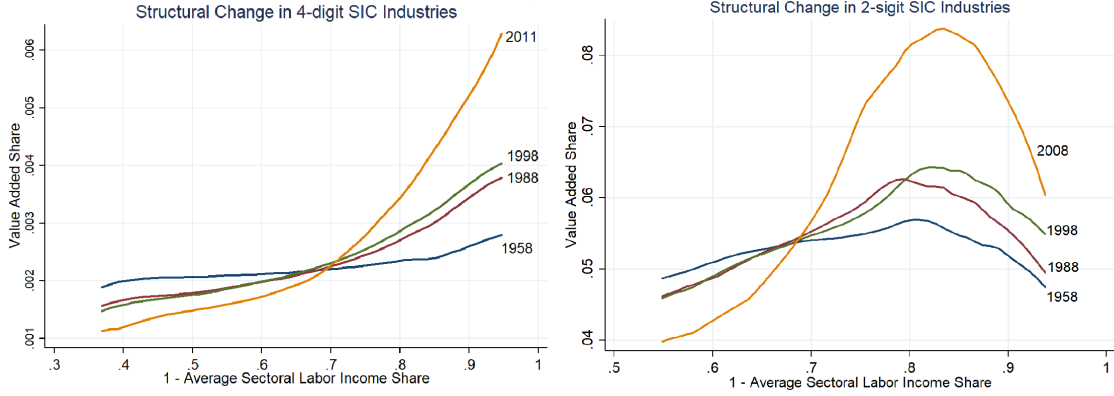


Figure 7: Structural Change in US

5 Simulation and Counter-factual Analysis

5.1 Simulation of the Model

In this section, we simulate our model, and conduct the counterfactual test of how the structural change would be affected by firms' technology choices. The counterfactual test would show the novelty of this paper to jointly consider structural change and technology choices. The simulation is based on calibration of exogenous parameters. First, we take into the parameters we estimated in the last section. Then the parameters to further calibrate include demand shifter θ_{it} for each industry i in year t and the constant ρ in the CES utility function. We assume the elasticity of substitution between differentiated products is 5 and calibrate ρ as 0.8.

In this section, we group all US manufacturing sectors to five groups, from labor intensive to capital intensive, based on the average capital intensity in the whole sample period. We plot the subsequent capital intensities of the five re-grouped industries in Figure 8. The figure shows that the capital intensity is increasing in each group of industries, and the relative order remain stable overtime. The pattern we show is consistent with the model prediction.

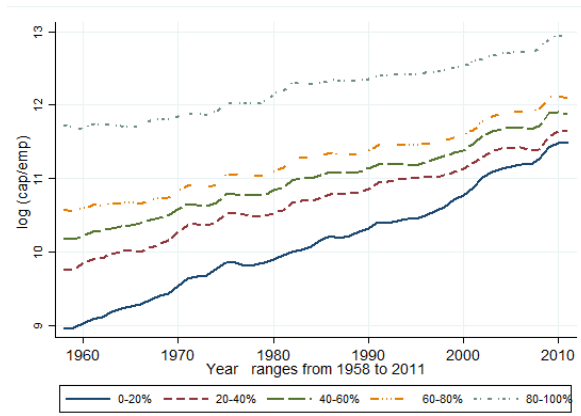


Figure 8: Capital Intensities in US

We estimated technology frontier using the same method from the last section, Figure 9 shows the changes in the $\gamma_{it} = m_{it}/n_{it}$. We can see that the technology frontier parameter γ_{it} was generally rising in each industry. The model predicts that, a rising γ_{it} induces increasing capital intensity, yet declining in a_{it}/b_{it} when capital and labor are complementary.

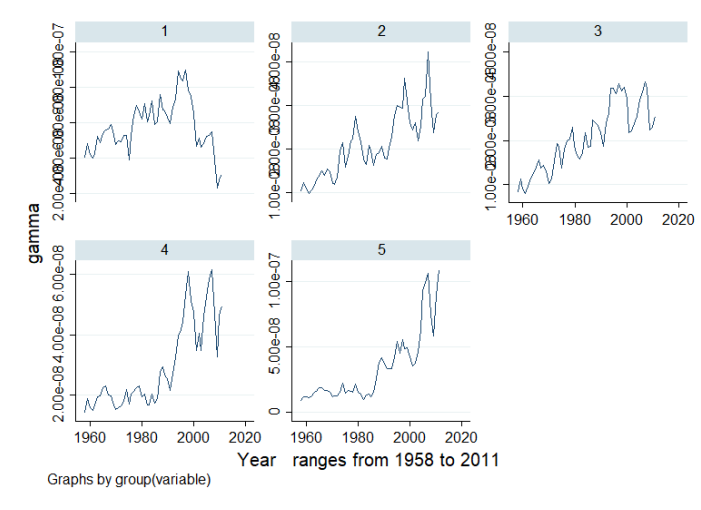


Figure 9: Estimated Frontier m_{it}/n_{it} in US

We then lay out the procedures to simulate the model.

Step 1:

The first step is to use the data of industrial value added share to back out the demand shifter θ_i for each sector, using equation from section 3.6. The calibrated demand shifters θ_{it} are shown in Figure 10. We find the demand shifters are relatively more stable than the technology frontiers, showing an declining pattern in industry 1 and 5, and increasing pattern in 2 and 3, stable but with some unstable change after 2000 in industry 4.



Figure 10: Calibrated Demand Shifter θ_{it} in US

Step 2:

In each counterfactual analysis, the factor price is solved by equation (2). It is worth noting that in the model we assume uniform factor price in all sectors, while in the data, each sector has heterogeneous factor price⁴.

Step 3:

Solve for counterfactual variables of capital, labor, value added share, using equation in section 3.6. The results are shown in the next subsection.

5.2 Industrial Structure without Technology Choices

Taking m and n in the first year as fixed in the whole sample period, and solve the model. Figure 11 shows that value added shares of each industry over time, where the blue line is the benchmark model, while the red line is the counterfactual model. It shows that the changes in industry 1-3 are similar between benchmark and the counterfactual models, while the increasing value added shares in industry 4 and 5 would be delayed when there were no technology progress. It implies that the consideration of technology choices are essential in explaining the structural change, since the changes in technology choices improves the optimality in the use of factors, thus accelerate the structural change.

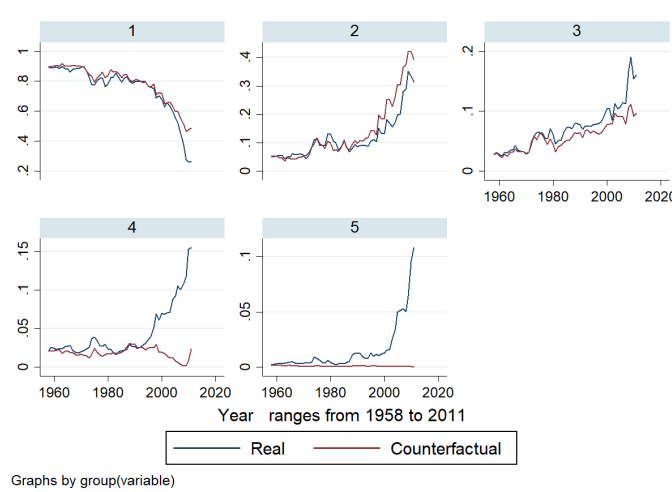


Figure 11: Counter-factual Structural Change

Figure 12 further shows an example of how the industrial structure is like in three different years, with the upper panel indicating the benchmark model and the lower panel as the counterfactual model. It shows that in both models, the industrial structure moves from labor intensive to capital intensive sectors. The counterfactual model shows slower rate in structural change.

⁴In the counterfactual analysis, we use the model predictions under baseline parameters, to compare with the model predictions under counterfactual parameters.

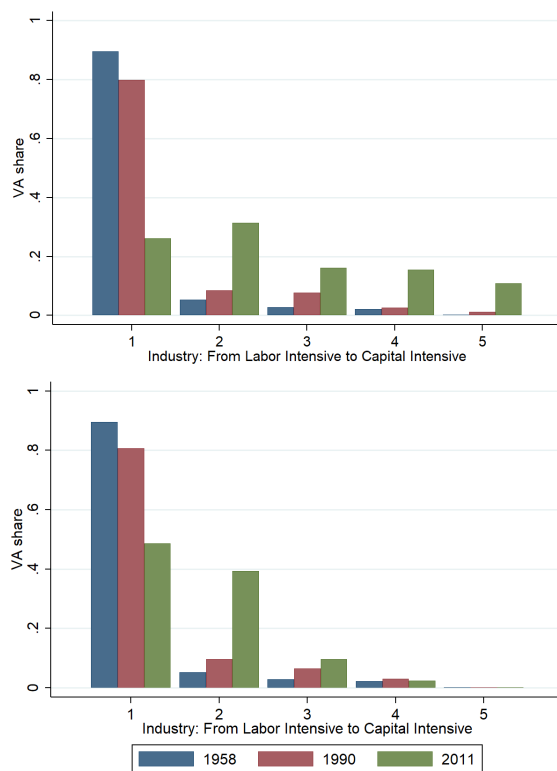


Figure 12: Counter-factual Structural Change: Three Years Example (upper: benchmark, lower: counterfactual)

6 Alternative approach

In the above analysis, one of the fundamental points is how to define the set of feasible technologies for choice.

To this end, we can modify the above model framework and take an alternative approach as follows.

There are only n industries, for any $i = 1, \dots, n$, the possible production functions of the industry i are

$$F_i(K, L) = K^{\alpha_i} (A_i L)^{\beta_i},$$

and

$$\alpha_i \in [\underline{\alpha}_i, \bar{\alpha}_i],$$

where $K \geq 0, L \geq 0$ are the physical capital and human capital, respectively, and $\beta_i = 1 - \alpha_i$, and $0 < \underline{\alpha}_i < \bar{\alpha}_i < 1$ and $A_i > 0$ are given, here A_i can be interpreted as labor augmenting factor.

And, for simplicity, we only consider a Cobb-Douglas type utility function, that is, we assume that the utility function of each consumer is

$$U(C_1, C_2, \dots, C_n) = \prod_{i=1}^n C_i^{\theta_i},$$

where $C_i \geq 0$ is the final good of the i -industry the consumer consumes, and $\theta_i \in (0, 1)$ is given, satisfying $\sum_{i=1}^n \theta_i = 1$.

And, moreover, we assume that in each industry, there are more than one but finite number of firms, and each firm can take only one technology. We notice that this assumption can be modified furthermore.

First of all, in each industry, facing any given price system (the product price $p > 0$, the capital price $r > 0$ and the labor price $\omega > 0$), if two firms take the same technology α , then, the capital-labor ratio's for them will be the same. In fact, if the optimization problem

$$\begin{aligned} \max \quad & pK^\alpha L^\beta - rK - \omega L, \\ \text{s.t.} \quad & K \geq 0, L \geq 0 \end{aligned}$$

has non-zero solution, then, the solution satisfies that

$$r = \alpha k^{-\beta},$$

and hence,

$$k = \left(\frac{\alpha}{r} \right)^{1/\beta},$$

where

$$k = \frac{K}{L}$$

is the capital-labor ratio.

Therefore, in one industry, any firms, each of which take the same technology, have the same capital-labor ratio, and hence can be "combined" into one firm, that is, if for any $i = 1, \dots, J$, where

J is some natural number, it holds that

$$Y_i = K_i^\alpha L_i^\beta, \quad k = \frac{K_i}{L_i},$$

where $k > 0$ is fixed, then,

$$Y = K^\alpha L^\beta,$$

where

$$Y = \sum_{i=1}^J Y_i, \quad K = \sum_{i=1}^J K_i, \quad L = \sum_{i=1}^J L_i.$$

In the sequel, we do not distinguish the following two cases: there is only one firm taking technology α and capital-labor ratio k and produce Y ; there are J firms, each of which takes the same technology α and capital-labor ratio k , and

$$Y = \sum_{i=1}^n Y_i,$$

where Y_i is the production of the i -firm. Put it another way, in fact, we consider a specific classification: in each industry, any two groups of firms can be said as equivalent, if all firms in these two groups take the same technology and capital-labor ratio, and the sums of productions in the two groups are equal to each other.

Secondly, by Lemma 2, we know that facing any fixed prices system, any firm will take the extreme technology, that is, either $\alpha = \underline{\alpha}$ or $\alpha = \bar{\alpha}$. Now, we consider an industry, in which there are two firms, taking technologies $\underline{\alpha}$ and $\bar{\alpha}$ respectively. Then, clearly, for any fixed $p > 0, r > 0, \omega > 0$, solving the optimization problem

$$\begin{aligned} \max \quad & p \left(K_1^{\underline{\alpha}} L_1^{\underline{\beta}} + K_2^{\bar{\alpha}} L_2^{\bar{\beta}} \right) - r(K_1 + K_2) - \omega(L_1 + L_2), \\ \text{s.t.} \quad & K_i \geq 0, L_i \geq 0, \quad i = 1, 2 \end{aligned}$$

is equivalent to solving the following two optimization problem simultaneously

$$\begin{aligned} \max \quad & p K_1^{\underline{\alpha}} L_1^{\underline{\beta}} - r K_1 - \omega L_1, \\ \text{s.t.} \quad & K_1 \geq 0, L_1 \geq 0, \end{aligned}$$

and

$$\begin{aligned} \max \quad & p K_2^{\bar{\alpha}} L_2^{\bar{\beta}} - r K_2 - \omega L_2, \\ \text{s.t.} \quad & K_2 \geq 0, L_2 \geq 0. \end{aligned}$$

And hence, the two firms can also be "combined" into one firm.

Based on the above consideration, we now give our modification in another equivalent way: in each industry, there is only one firm, but it can take a "mixed" technology, that is, it can take the two extreme technologies simultaneously, all else remain unchanged.

Notice that, in fact, in this case, for any $i = 1, \dots, n$, in the i -industry, the total production

possibility set \mathcal{Y}'_i is the sum of the production possibility sets for each technology, which is the union of the sum of any finite number of the production possibility sets for each technology. Here, the meaning of "sum" of any finite number of sets is as follows: for any sets S_1, \dots, S_n in some same Euclidian space, where n is a natural number,

$$\sum_{i=1}^n S_i =: \left\{ \sum_{i=1}^n s_i \mid s_i \in S_i, \quad i = 1, \dots, n \right\}.$$

Clearly, if the "mixed" technology is not allowed, that is, an firm can only take one technology (we say it takes only pure technology), then, in the i -industry, the total production possibility set \mathcal{Y}_i in that case is just the union of the production possibility sets for each technology.

It's easy to see that \mathcal{Y}'_i is just the convex hull of \mathcal{Y}_i and hence convex, although \mathcal{Y}_i is not convex. And, the efficient frontier for \mathcal{Y}'_i is just the set of $(Y, -K, -L)$ satisfying $K \geq 0, L \geq 0$, and

$$Y = \max \left(K_1^{\alpha_i} L_1^{\beta_i} + K_2^{\bar{\alpha}_i} L_2^{\bar{\beta}_i} \right),$$

s.t.

$$\begin{aligned} K_1 + K_2 &= K, \quad L_1 + L_2 = L, \\ K_i &\geq 0, \quad L_i \geq 0, \quad i = 1, 2. \end{aligned}$$

By the way, it's easy to see that for the last optimization problem, suppose that $K > 0, L > 0$, and denote $k = K/L$, and denote the solution of this problem as $K_1^*, K_2^*, L_1^*, L_2^*$, then,

$$K_1^* > 0, L_1^* > 0, K_2^* = 0, L_2^* = 0,$$

if and only if

$$k \leq \underline{B}_i;$$

and

$$K_1^* > 0, L_1^* > 0, K_2^* > 0, L_2^* > 0,$$

if and only if

$$\underline{B}_i < k < \bar{B}_i;$$

and

$$K_1^* = 0, L_1^* = 0, K_2^* > 0, L_2^* > 0,$$

if and only if

$$k \geq \bar{B}_i,$$

where

$$\begin{aligned} \underline{B}_i &= \left[\left(\frac{\alpha_i}{\bar{\alpha}_i} \right)^{\bar{\alpha}_i} \left(\frac{\beta_i}{\bar{\beta}_i} \right)^{\bar{\beta}_i} \right]^{\frac{1}{\bar{\alpha}_i - \alpha_i}}, \\ \bar{B}_i &= \left[\left(\frac{\alpha_i}{\bar{\alpha}_i} \right)^{\alpha_i} \left(\frac{\beta_i}{\bar{\beta}_i} \right)^{\beta_i} \right]^{\frac{1}{\bar{\alpha}_i - \alpha_i}}. \end{aligned}$$

That is, when and only when the capital-labor ratio is allocated in some interval, not too large and not too small, the firm will take a mixed technology.

As to the two quantities \underline{B}_i and \overline{B}_i , we point out that, in fact, in the $k - y$ plane, \underline{B}_i is just the k -coordinate of the tangency point of the curve $y = k^{\alpha_i}$ and the line L , and \overline{B}_i is the k -coordinate of the tangency point of the curve $y = k^{\overline{\alpha}_i}$ and the line L , where L is the joint tangent line of the above two curves.

6.1 General equilibrium and its solution

Now, we give our formal definition of general equilibrium in this setup as follows.

Definition 2. $(C_i^*, K_{ij}^*, L_{ij}^*, p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$ is an equilibrium, where, for any i, j ,

$$\begin{aligned} C_i^* &\geq 0, \\ K_{ij}^* &\geq 0, \quad L_{ij}^* \geq 0, \\ p_i^* &\geq 0, \quad r^* \geq 0, \quad \omega^* \geq 0, \end{aligned}$$

if

(I)

$$(C_1^*, \dots, C_n^*) \in \arg \max_{C_1, C_2} U(C_1, \dots, C_n),$$

s.t.

$$\begin{aligned} \sum_{i=1}^n p_i^* C_i &\leq r^* K_0 + \omega^* L_0, \\ C_i &\geq 0, \quad i = 1, \dots, n; \end{aligned}$$

(II) for any $i = 1, 2$,

$$\begin{aligned} &(K_{i1}^*, K_{i2}^*, L_{i1}^*, L_{i2}^*) \\ \in &\arg \max_{K_{i1}, K_{i2}, L_{i1}, L_{i2}} \left\{ p_i^* \left(K_{i1}^{\alpha_i} (A_i L_{i1})^{\beta_i} + K_{i2}^{\overline{\alpha}_i} (A_i L_{i2})^{\overline{\beta}_i} \right) - r^* (K_{i1} + K_{i2}) - \omega^* (L_{i1} + L_{i2}) \right\}, \end{aligned}$$

s.t.

$$K_{ij} \geq 0, \quad L_{ij} \geq 0, \quad j = 1, 2;$$

(III)

$$\begin{aligned} C_i^* &= (K_{i1}^*)^{\alpha_i} (A_i L_{i1}^*)^{\beta_i} + (K_{i2}^*)^{\overline{\alpha}_i} (A_i L_{i2}^*)^{\overline{\beta}_i}, \quad i = 1, 2, \dots, n, \\ \sum_{i,j} K_{ij}^* &= K_0, \\ \sum_{i,j} L_{ij}^* &= L_0. \end{aligned}$$

Remark 6. One can see that in this setup, if $(C_i^*, K_{ij}^*, L_{ij}^*, p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$ is an equilibrium, then, for any i ,

$$C_i^* > 0, \quad p_i^* > 0, \quad r^* > 0, \quad \omega^* > 0,$$

and for any i, j , $K_{ij} = 0$ if and only if $L_{ij} = 0$.

For any $i = 1, \dots, n$, we denote

$$\tau_i = \left(\frac{\underline{\alpha}_i^{\alpha_i} \underline{\beta}_i^{\beta_i}}{\bar{\alpha}_i^{\alpha_i} \bar{\beta}_i^{\beta_i}} \right)^{1/(\bar{\alpha}_i - \underline{\alpha}_i)}, \quad \sigma_i = A_i \tau_i.$$

We arrange $\sigma_1, \dots, \sigma_n$ in increasing order. For simplicity of the notation, without any loss of generality, we assume

$$\sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_n.$$

We then rank the industries accordingly.

Now, we introduce our key notations. For any $i = 1, 2, \dots, n$, denote

$$k_{2i-1}^* = \frac{\sum_{j < i} \theta_j \bar{\alpha}_j + \sum_{j \geq i} \theta_j \underline{\alpha}_j}{\sum_{j < i} \theta_j \bar{\beta}_j + \sum_{j \geq i} \theta_j \underline{\beta}_j} \sigma_i.$$

$$k_{2i}^* = \frac{\sum_{j \leq i} \theta_j \bar{\alpha}_j + \sum_{j > i} \theta_j \underline{\alpha}_j}{\sum_{j \leq i} \theta_j \bar{\beta}_j + \sum_{j > i} \theta_j \underline{\beta}_j} \sigma_i.$$

Clearly,

$$0 < k_1^* \leq k_2^* \leq k_3^* \leq \dots \leq k_{2n}^* < \infty.$$

For simplicity, we denote $k_0^* = 0$, $k_{2n+1}^* = \infty$, and $\sigma_0 = 0$, $\sigma_{2n+1} = \infty$.

For convenience, we introduce some further notations. For industry $i = 1, \dots, n$, if the firm i takes the low technology $\underline{\alpha}_i$, then, we denote this matter as \underline{i} ; if it takes the high technology $\bar{\alpha}_i$, we denote it as \bar{i} ; if it takes both technologies, or put it another way, it takes a mixed technology, then, we denote it as i . Then, firm i has three choices: $\underline{i}, i, \bar{i}$. Clearly, there are totally 3^n cases for the combination of the n industries to choose their technologies, and at least one happens, and this comes from the classical Arrow-Debreu general equilibrium existence theorem.

However, somewhat surprisingly, among the 3^n cases, there are in fact only $2n + 1$ cases happen.

Theorem 3. For any $k_0 > 0$, there exists unique equilibrium $(C_i^*; K_{ij}^*, L_{ij}^*; p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$, the solution of which is determined as follows. For this $k_0 > 0$, there exists unique $t \in \{0, 1, \dots, n\}$ such that either

$$k_{2t}^* < k_0 < k_{2t+1}^*,$$

or

$$k_{2t-1}^* \leq k_0 \leq k_{2t}^*.$$

(i) If

$$k_{2t}^* < k_0 < k_{2t+1}^*,$$

then, for any $i \leq t$, happens \bar{i} ; for any $i > t$, happens \underline{i} , and

$$\begin{aligned}
K_{i1}^* &= 0, \quad K_{i2}^* = \frac{\theta_i \bar{\alpha}_i}{\sum_{j \leq t} \theta_j \bar{\alpha}_j + \sum_{j > t} \theta_j \underline{\alpha}_j} K_0, \quad \forall i \leq t, \\
K_{i1}^* &= \frac{\theta_i \underline{\alpha}_i}{\sum_{j \leq t} \theta_j \bar{\alpha}_j + \sum_{j > t} \theta_j \underline{\alpha}_j} K_0, \quad K_{i2}^* = 0, \quad \forall i > t; \\
L_{i1}^* &= 0, \quad L_{i2}^* = \frac{\theta_i \bar{\beta}_i}{\sum_{j \leq t} \theta_j \bar{\beta}_j + \sum_{j > t} \theta_j \underline{\beta}_j} L_0, \quad \forall i \leq t, \\
L_{i1}^* &= \frac{\theta_i \underline{\beta}_i}{\sum_{j \leq t} \theta_j \bar{\beta}_j + \sum_{j > t} \theta_j \underline{\beta}_j} L_0, \quad K_{i2}^* = 0, \quad \forall i > t;
\end{aligned}$$

The prices are determined by the following relations:

$$\begin{aligned}
\frac{r^*}{p_i^*} &= \underline{\alpha}_i g_{i1}^{-\underline{\beta}_i}, \quad \frac{\omega^*}{p_i^*} = A_i \underline{\beta}_i g_{i1}^{\underline{\alpha}_i}, \quad \forall i \leq t, \\
\frac{r^*}{p_i^*} &= \bar{\alpha}_i g_{i2}^{-\bar{\beta}_i}, \quad \frac{\omega^*}{p_i^*} = A_i \bar{\beta}_i g_{i2}^{\bar{\alpha}_i}, \quad \forall i > t.
\end{aligned}$$

(ii) If

$$k_{2t-1}^* \leq k_0 \leq k_{2t}^*,$$

then, for any $i < t$, happens \bar{i} ; for $i = t$, happens i ; for any $i > t$, happens \underline{i} , and

$$\begin{aligned}
K_{i1}^* &= 0, \quad K_{i2}^* = \theta_i \bar{\alpha}_i (K_0 + \sigma_t L_0), \quad \forall i < t, \\
K_{i1}^* &= \theta_i \underline{\alpha}_i (K_0 + \sigma_t L_0), \quad K_{i2}^* = 0, \quad \forall i > t, \\
K_{t1}^* &= \frac{\sum_{j \leq t} \theta_j \bar{\beta}_j + \sum_{j > t} \theta_j \underline{\beta}_j}{\bar{\alpha}_t / \underline{\alpha}_t - 1} \left(\frac{k_{2t}^*}{k_0} - 1 \right) K_0, \\
K_{t2}^* &= \frac{\sum_{j < t} \theta_j \bar{\beta}_j + \sum_{j \geq t} \theta_j \underline{\beta}_j}{1 - \underline{\alpha}_t / \bar{\alpha}_t} \left(1 - \frac{k_{2t-1}^*}{k_0} \right) K_0, \\
L_{t1}^* &= \frac{\sum_{j \leq t} \theta_j \bar{\alpha}_j + \sum_{j > t} \theta_j \underline{\alpha}_j}{1 - \bar{\beta}_t / \underline{\beta}_t} \left(1 - \frac{k_0}{k_{2t}^*} \right) L_0, \\
L_{t2}^* &= \frac{\sum_{j < t} \theta_j \bar{\alpha}_j + \sum_{j \geq t} \theta_j \underline{\alpha}_j}{\underline{\beta}_t / \bar{\beta}_t - 1} \left(\frac{k_0}{k_{2t-1}^*} - 1 \right) L_0.
\end{aligned}$$

The prices are determined by the following relations:

$$\begin{aligned}
\frac{r^*}{p_i^*} &= \underline{\alpha}_i g_{i1}^{-\underline{\beta}_i}, \quad \frac{\omega^*}{p_i^*} = A_i \underline{\beta}_i g_{i1}^{\underline{\alpha}_i}, \quad \forall i \leq t, \\
\frac{r^*}{p_i^*} &= \bar{\alpha}_i g_{i2}^{-\bar{\beta}_i}, \quad \frac{\omega^*}{p_i^*} = A_i \bar{\beta}_i g_{i2}^{\bar{\alpha}_i}, \quad \forall i \geq t.
\end{aligned}$$

And in all of the above two cases,

$$C_i^* = (K_{i1}^*)^{\alpha_i} (A_i L_{i1}^*)^{\beta_i} + (K_{i2}^*)^{\bar{\alpha}_i} (A_i L_{i2}^*)^{\bar{\beta}_i}, \quad \forall i = 1, \dots, n,$$

where

$$k_{ij} = \frac{K_{ij}^*}{L_{ij}^*}, \quad g_{ij} = k_{ij}/A_i, \quad \forall i = 1, \dots, n, j = 1, 2.$$

From this theorem, we get the following corollary about the pattern of the change of the relative price of labor over capital along with the change of k_0 .

Corollary 7. With respect to k_0 , the equilibrium relative price of labor over capital ω^*/r^* is increasing, continuous and piecewise linear, the graph of which is a fold line, more precisely,

$$\frac{\omega^*}{r^*} = \begin{cases} \frac{\sigma_t}{k_{2t-1}^*} k_0, & k_{2t-2}^* < k_0 < k_{2t-1}^*, \quad t = 1, \dots, n, \\ \sigma_t, & k_{2t-1}^* \leq k_0 \leq k_{2t}^*, \quad t = 1, \dots, n, \\ \frac{\sigma_n}{k_{2n}^*} k_0, & k_0 > k_{2n}^*. \end{cases}$$

From this theorem, we get the following proposition about the technology upgrading, which is similar to the Proposition 5 in the above model framework.

Proposition 6. (Technology upgrading) In the alternative model framework, along with the increase of capital per capita of the society, each industry experience three phases of technology upgrading: low technology, mixed technology and high technology technology; the industries take technology upgrading sequentially, one industry starts to upgrade, after its previous industry has completed its upgrading; and the economy as a whole experiences $2n + 1$ phases of technology upgrading.

Now, we consider the social planner's problem. In this setup, the social planner's problem can be stated as follows:

$$\begin{aligned} \max \quad & \prod_{i=1}^n C_i^{\theta_i}, \\ \text{s.t.} \quad & C_i = K_{i1}^{\alpha_i} (A_i L_{i1})^{\beta_i} + K_{i2}^{\bar{\alpha}_i} (A_i L_{i2})^{\bar{\beta}_i}, \quad i = 1, \dots, n, \\ & \sum_{ij} K_{ij} = K_0, \quad \sum_{ij} L_{ij} = L_0, \\ & K_{ij} \geq 0, \quad L_{ij} \geq 0, \quad i = 1, \dots, n, j = 1, 2. \end{aligned}$$

One can prove that this social planner's problem has a unique solution in any case.

Remark 7. If any firm is allowed to take only pure technology but not mixed technology, then, from theorem 3, we know that the equilibrium may not exist for some structure of factor endowment. But, even in the case, where only pure technology is allowed, the social planner's problem has (unique) solution, and of course, its outcome is Pareto efficient. Therefore, in this case, the second welfare theorem does not hold, that is, there is a Pareto efficient allocation which can not be achieved by the market competition.

We give an example.

Example. Let $n = 2, A_1 = A_2 = 1$, and

$$\theta_1 = \theta_2 = 1/2,$$

$$[\underline{\alpha}_1, \bar{\alpha}_1] = [1/4, 2/4],$$

$$[\underline{\alpha}_2, \bar{\alpha}_2] = [2/4, 3/4].$$

In this case, we have that

$$\tau_1 = \frac{27}{16}, \quad \tau_2 = \frac{16}{27},$$

and

$$k_1^* = \frac{16}{45}, \quad k_2^* = \frac{16}{27}, \quad k_3^* = \frac{27}{16}, \quad k_4^* = \frac{45}{16}.$$

And hence, if

$$k_0 \leq 16/45,$$

or

$$k_0 \in [16/27, 27/16],$$

or

$$k_0 \geq 45/16,$$

then any firm takes pure technology, and (α_1^*, α_2^*) is equal to

$$(1/4, 2/4),$$

or

$$(1/4, 3/4),$$

or

$$(2/4, 3/4),$$

respectively.

If

$$k_0 \in (16/45, 16/27)$$

or

$$k_0 \in (27/16, 45/16),$$

then at least one firm takes mixed technology. And hence, if only pure technology is allowed, then, in this case, the equilibrium does not exist.

Clearly, in the case, where mixed technologies are allowed, the economy will experience 5 phases of technology upgrading along with the increase of k_0 .

6.2 Extreme case

In the above setting, we avoid the cases, where the α_i 's can touch 0 or 1, that is, the technology relies only on labor or capital. In this subsection, we consider the extreme case, where

$$\alpha_i \in [0, 1], \quad i = 1, \dots, n,$$

that is, by the notations used above, for any $i = 1, \dots, n$,

$$\underline{\alpha}_i = 0, \quad \bar{\alpha}_i = 1.$$

For simplicity, we assume that $A_i = 1$ for all i .

In this setting, we know that each firm only takes the extreme technologies. And hence, for simplicity, we say $(C_i^*; K_i^*, L_i^*; p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$ is an equilibrium, if $(C_i^*; K_{ij}^*, L_{ij}^*; p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$ is an equilibrium by the above standard notation, where $K_{i1} = 0 = L_{i2}$, and $K_{i2} = K_i$, $L_{i1} = L_i$.

Theorem 4. There exists multiple equilibria. And, $(C_i^*; K_i^*, L_i^*; p_i^*, r^*, \omega^*)_{i=1, \dots, n, j=1, 2}$, is an equilibrium, if and only if

$$\begin{aligned} C_i^* &= K_i^* + L_i^* = \theta_i(K_0 + L_0), \\ K_0 &= \sum_{i=1}^n K_i^*, \\ L_0 &= \sum_{i=1}^n L_i^* \\ r^* &= \omega^* = p_i^* > 0, \quad \forall i. \end{aligned}$$

Corollary 8. In this extreme setting, if only pure technologies are allowed, then, the equilibrium exists, if and only if there exists a subset $J \subset \{1, \dots, n\}$ such that

$$k_0 = \frac{\sum_{i \notin J} \theta_i}{\sum_{i \in J} \theta_i}.$$

For example, if $n = 2$, then, if only pure technologies are allowed, then, then equilibrium equilibrium exists, if and only if $k_0 = \theta_2/\theta_1$ or $k_0 = \theta_1/\theta_2$. And hence, the cases of equilibrium existence are rare.

The extreme case in this subsection is not interesting in the sense that the equilibrium prices are all equal, and hence, we can not investigate the relative price change along with the change of the endowment structure. But, we put it here as a contrast to the above formal setting. By this contrast, one can see why we set up our main model framework as above. In addition, the extreme technologies do not coincide with the reality, in any industry nowadays in the real world uses only labor or only capital.

7 Extension to Dynamic model

Following the idea of Ju. et al (2015), we can extend the above static model to dynamic one, through which we will see that the capital per capita is increasing along with time going, and then, with the help of the comparative static analysis above, we can characterize the industrial dynamics along with the time passing.

7.1 Basic model

We now consider a closed economy, which exists on the whole time interval $[0, \infty)$. The population keeps constant, more concretely, there are finite number of homogeneous individuals, each is alive on the whole time interval $[0, \infty)$, owning initial endowments at time $t = 0$ the physical capital $Z_0 > 0$ and labor $L_0 = 1$, with life long utility function

$$U = \int_0^\infty e^{-\delta t} u(C(t)) dt,$$

where $\delta > 0$ is his time discount rate, and $C(t)$ is his consumption at time t , and

$$u(C) = \frac{C^{1-\theta} - 1}{1-\theta},$$

where $\theta \in (0, 1]$.

We assume that there is only one consumption good, and at any time point t , in this economy, there are three types of sectors. The first type: there are continuum of sectors on the interval $[0, 1]$, each producing intermediate goods by use of capital and labor; the second type: there is only one sector, producing the final consumption good by use of all the intermediate goods; the third type: there is only one sector, producing capital good by use of capital only.

The production in each sector is as follows. For the first type sector, the production is set up as in 3.1. For the second type sector, the production function is

$$C = \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho},$$

where $\rho \in (0, 1)$ as in 3.1, and C_i is the intermediate good i , and C is the final consumption good. This is just the welfare function set up in 3.1. The setup here can be seen as equivalent to that in 3.1 in the sense that we can interpret the welfare in the setup in 3.1 as the final good.

For the third type sector, the production is proceeded by an AK technology, which itself comes from the effect of learning by doing.

And, at any time, each capital owner will take out some of his capital to sell to the second type sectors in the capital market, and sell his labor to the second type sectors in the labor market, and earn his income to buy the final consumption good to support his living.

We emphasize that in this setting, the capital good is different from other goods, including the intermediate goods and the final consumption good. The capital good can not be used in the production of the final good, and is not consumed like the final consumption good. For the individual the capital is only a tool to get income. None of intermediate goods and the final good can be accumulated, they

are used up for production or consumed up once they are produced.

And hence, the capital stock movement equation of this economy is as follows:

$$\dot{Z}(t) = aZ(t) - K(t), \quad \forall t \geq 0,$$

where $Z(t)$ is the capital stock at time t , and $a > 0$ is a constant, and $K(t)$ is the working capital at time t , which is supplied to the second type sectors. We assume that $a > \delta$.

As usual, we assume that all markets are competitive, and all firms can survive only instantly. Now, we can define the dynamic equilibrium in the standard way. For simplicity, without any loss of generality, we may assume that the number of individuals is 1, and the his labor is normalized to be 1. And, we assume that the price of the final good is 1.

We make a convention as usual that all variables here are nonnegative, and for simplicity, in any optimization problems, the nonnegative constraints are omitted to write out explicitly.

Definition 3. $(p_i^*(t), r^*(t), \omega^*(t), a_i^*(t), b_i^*(t), K_i^*(t), L_i^*(t), C_i^*(t), C^*(t), K^*(t), Z^*(t))_{i \in [0,1], t \geq 0}$ is called an (dynamic) equilibrium, if

$$(Z^*, K^*, C^*) \in \arg \max_{Z, K, C} \int_0^\infty e^{-\delta t} u(C(t)) dt,$$

s.t.

$$\begin{aligned} \dot{Z}(t) &= aZ(t) - K(t), \\ C(t) &\leq r(t)K(t) + \omega(t), \quad \forall t, \\ Z(0) &= Z_0; \end{aligned}$$

and for any $i \in [0, 1]$ and any $t \geq 0$,

$$\begin{aligned} &(a_i^*(t), b_i^*(t), K_i^*(t), L_i^*(t)) \\ &\in \arg \max_{a, b, K, L} \left\{ p_i^*(t) ((aK)^{\rho_i} + (bL)^{\rho_i})^{1/\rho_i} - r^*(t)K - \omega^*(t)L \right\}, \end{aligned}$$

s.t.

$$\left(\frac{a}{m_i} \right)^{\sigma_i} + \left(\frac{b}{n_i} \right)^{\sigma_i} = 1;$$

and for any $t \geq 0$,

$$(C_i^*(t))_{i \in [0,1]} \in \arg \max_{(C_i)_{i \in [0,1]}} \left\{ \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho} - \int_0^1 p_i^*(t) C_i di \right\};$$

and

$$\begin{aligned}
C_i^*(t) &= ((a_i^*(t)K_i^*(t))^{\rho_i} + (b_i^*(t)L_i^*(t))^{\rho_i})^{1/\rho_i}, \quad \forall i, t, \\
C^*(t) &= \left(\int_0^1 \theta_i (C_i^*(t))^{\rho} di \right)^{1/\rho}, \quad \forall t, \\
\int_0^1 K_i^*(t) di &= K^*(t), \quad \forall t, \\
\int_0^1 L_i^*(t) di &= 1, \quad \forall t.
\end{aligned}$$

Here, all the parameters $\rho, \rho_i, \sigma_i, m_i, n_i$ are as in 3.1. From this definition, one can easily see that all prices are strictly positive (in the sense of Lebesgue measure). We can prove that the equilibrium exists and is unique in the sense of Lebesgue measure.

It's worth mentioning that there is a little bit difference between this setting and the setting in 3.1 for the treatment of production. In 3.1, only a proportion between any two different prices is determined, but here, since we set the price of the final good to be 1, and hence, all other prices will be determined uniquely.

The social planner's problem in this setting is

$$\begin{aligned}
\max \quad & \int_0^\infty e^{-\delta t} u(C) dt, \\
\text{s.t.} \quad & \dot{Z} = aZ - K, \\
& C = \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho}, \\
& C_i = ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i}, \quad i \in [0, 1] \\
& (a_i/m_i)^{\sigma_i} + (b_i/n_i)^{\sigma_i} = 1, \quad i \in [0, 1], \\
& \int_0^1 K_i di = K, \quad \int_0^1 L_i di = 1, \\
& Z(0) = Z_0.
\end{aligned}$$

Here, for simplicity, we omit to write out (t) for all dynamic variables, for example, in the above dynamic optimization problem, Z is $Z(t)$, K_i is $K_i(t)$, etc. In the sequel, we will use these simple notations, if only it does not induce any confusion, and for any differential equation, we omit to write out $\forall t \geq 0$.

It's easy to see that solving this problem is equivalent to solving the following two problems sequentially. First step, solve the problem \mathbb{P}_1

$$\begin{aligned}
C = W(K) = \max \quad & \left(\int_0^1 \theta_i C_i^\rho di \right)^{1/\rho}, \\
\text{s.t.} \quad & C_i = ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i}, \quad i \in [0, 1] \\
& (a_i/m_i)^{\sigma_i} + (b_i/n_i)^{\sigma_i} = 1, \quad i \in [0, 1], \\
& \int_0^1 K_i di = K, \quad \int_0^1 L_i di = 1;
\end{aligned}$$

then, second step, solve the problem \mathbb{P}_2 :

$$\begin{aligned} \max \quad & \int_0^\infty e^{-\delta t} u(W(K)) dt, \\ \text{s.t.} \quad & \dot{Z} = aZ - K, \\ & Z(0) = Z_0. \end{aligned}$$

The first problem \mathbb{P}_1 is just the social planner's problem in the static model as in 3.4, from there we know that

$$W(K) = \left(\frac{\int_0^1 \theta_i^{1/(1-\rho)} (n_i(1 + (\gamma_i z)^{\delta_i})^{1/\delta_i})^{\rho/(1-\rho)} di}{\int_0^1 [\theta_i n_i^\rho (1 + (\gamma_i z)^{\delta_i})^{\rho/\varepsilon_i - 1}]^{1/(1-\rho)} di} \right)^{1/\rho},$$

and z is determined by the equation

$$K = \frac{z^{1/(1-\rho)} \int_0^1 [\theta_i m_i^\rho (1 + (\gamma_i z)^{-\delta_i})^{\rho/\varepsilon_i - 1}]^{1/(1-\rho)} di}{\int_0^1 [\theta_j n_j^\rho (1 + (\gamma_j z)^{\delta_j})^{\rho/\varepsilon_j - 1}]^{1/(1-\rho)} dj}.$$

One can verify that W is a smooth function, satisfying $W(0) = 1^5$, $W(\infty) = \infty$, and for any $K > 0$, $W'(K) > 0$, $W''(K) < 0$, that is, W is strictly increasing and strictly convex, and $W'(0) = \infty$, $W'(\infty) = 0$.

One can verify directly that the outcome of the dynamic equilibrium is just the solution of the social planner problem. Therefore, in our setting, the first welfare theorem still holds. That is, the market competition leads to a Pareto efficient allocation.

Let's look at the social planner problem \mathbb{P}_2 in more detail. It's a standard optimal control problem. Obviously, the Mangasarian sufficiency condition is satisfied, and all variables are non-negative, and of course, the linear independence constraint qualification is also satisfied, that is, this problem is a regular problem, and hence, if we denote this problem's current value Hamiltonian function as

$$H = u(W(K)) + \lambda(aZ - K),$$

then, $(K(t), Z(t))_{t \geq 0}$ is the solution of this problem, if and only if there exists a continuous and piecewise smooth $(\lambda(t))_{t \geq 0}$ such that

$$0 = H_K = C^{-\theta} W'(K) - \lambda, \tag{8}$$

$$-\dot{\lambda} + \delta \lambda = H_Z = a\lambda, \tag{9}$$

and TVC,

$$\lim_{t \rightarrow \infty} e^{-\delta t} \lambda(t) Z(t) = 0.$$

⁵This might be interesting, because with no working capital, the final good is not 0. However, this is not surprising, since in the ordinary CES function, capital and labor are substitutable, although imperfectly, the production can be proceeded, even if there is no capital, because labor exists and can be used always.

From (8), we have that $\lambda(t) > 0$, for any $t \geq 0$, and from (9), we get that the growth rate of λ is

$$\dot{\lambda} = \delta - a. \quad (10)$$

Here, $\dot{\lambda} = \dot{\lambda}/\lambda$. In the sequel, for any positive dynamic variable, we use the notation $\dot{}$ to denote its growth rate.

It's well known that at any time point $t \geq 0$, $\lambda(t)$ is just the shadow price of capital stock $Z(t)$. Note that there is no market for capital stock, therefore, there is no market price for it. Its shadow price is the representation of its true value to the economy. This shadow price is declining to 0 exponentially, which means that the capital is becoming less and less scarce. And, correspondingly, the capital stock is increasing continuously and monotonically to infinity.

Also from (8),(9), we can get the movement equation of K :

$$\dot{K} = \frac{a - \delta}{\frac{\theta}{W(K)} - \frac{W''(K)}{W'(K)}}.$$

And hence, $\dot{K} > 0$ for all $t \geq 0$. Therefore, along the equilibrium path, the working capital is strictly increasing. And, without any difficulty, one can prove that $K(t) \rightarrow \infty$ as $t \rightarrow \infty$. Since $C = W(K)$, and hence, the final consumption good is strictly increasing to the infinity also, along the equilibrium path. Therefore, this economy has no steady state and grows up forever.

From the TVC and the movement equation of the capital stock $\dot{Z} = aZ - K$ and the initial value condition $Z(0) = Z_0$, we obtain

$$Z_0 = \int_0^\infty e^{-at} K(t) dt.$$

This says that taking the "intrinsic growth rate" of the capital stock as the discounting rate, the present value of the working capital flow is just the initial capital stock. In other words, the initial capital stock, operated well by the individual, can just support the demand of working capital, and will be used up finally. And, this relationship does not concern with the social discount rate at all.

Just as in 3.1, one can see that there exist smooth decreasing function f and increasing function g such that along the dynamic equilibrium path, it holds that $r = f(K)$, $\omega = g(K)$. Then, $\dot{r} = f'(K)\dot{K} < 0$, $\dot{\omega} = g'(K)\dot{K} > 0$. Therefore, the real market price of working capital is decreasing, the market price of labor is increasing.

Noticing that $C = rK + \omega$, and hence,

$$W(K) = f(K)K + g(K). \quad (11)$$

Denote $\kappa = W'(K)$. This κ is just the shadow price of working capital K . We have that $\dot{\kappa} = W''(K)\dot{K} < 0$, and hence, the shadow price of the working capital is decreasing, and hence, the working capital, just like the capital stock, becomes less and less scarce.

We sum up the above analysis and give a proposition.

Proposition 7. Along the equilibrium path, the the capital stock, the working capital and the final good are all strictly increasing to infinity, and the economy grows up forever; the real market price of working capital is decreasing, the market price of labor is increasing; the shadow prices of the capital stock and the working capital are all decreasing to 0.

From (8),(10), we get the following proposition.

Proposition 8. (One version of the Keynes-Ramsey rule). Along the equilibrium path, the growth rate of final consumption good should satisfy

$$\dot{C} = \frac{1}{\theta} (\dot{\kappa} - \dot{\lambda}).$$

That is, the growth rate of consumption good is difference between the growth rates of the shadow prices of working capital and capital stock, divided by the consumer's Arrow-Pratt relative risk aversion.

Noticing that $\dot{C} > 0$, then from the above proposition, we get that

$$0 > \dot{\kappa} > \dot{\lambda}.$$

That is, the shadow price of the capital stock decreases more sharply than that of working capital, so that comparing with the capital stock, the working capital is more scarce, although both of them become less and less scarce.

From (11), it follows that

$$\kappa = r + \Delta,$$

where $\Delta = f'(K)K + g'(K)$, which is the difference between the shadow price and the real market price of the working capital.

We are interested in the sign of Δ , and even wonder if it holds that $\Delta = 0$. This problem is quite complicated. Along the equilibrium path, maybe, the sign of Δ changes. Intuitively, there exists some threshold K^* such that $\Delta(K^*) = 0$, and on the two sides of this threshold, Δ changes its sign, maybe from negative to positive, or from positive to negative. $\Delta > 0$ means the market encourages the individual to sell more working capital to the market; $\Delta < 0$ means the market restrains the individual to sell more working capital to the market.

Summing up the above analysis, we get the final result.

Proposition 9. Driven by the market competition, and by the "learning by doing" of the individuals, the economy grows up forever. Along this process, each industry in the second type sectors carries on continuously the technology upgrading, and the whole economy tends more and more to capital intensive industries.

7.2 Modified model

In the above model, the technology frontier for any industry in the second type sectors is fixed. In fact, we can let them change, either exogenously or endogenously.

If we assume that m_i, n_i all increase exogenously, along time going, the corresponding technology frontiers are extended outward. Then, all the treatment above works still, no any other technique is needed. We omit it.

If we consider R&D, the social planner's problem can be written out easily, as in the usual way. But, in our setting, if we want to find the micro-foundation for the R&D, in other words, we want to

set up a micro-model to depict this economy, then, the problem will become quite complicated. This will be our further work.

In addition, in fact, the static model in section 6 can also be extended to dynamic one, using the same method here. And, in that alternative approach, the process of technology upgrading is more clear, the industries take their technology upgrading sequentially, one by one, one starts immediately, once its previous industry has completed its technology upgrading, and any industry experiences three phases of technology upgrading from low technology, mixed technology to high technology, and totally, the economy experiences $2n+1$ phases, and finally, all industries complete their technology upgrading. The technology upgrading happens along with the increase of the working capital. And, just as in the above, we can prove that in this alternative approach, the capital stock and working capital are all increasing monotonically to infinity.

We omit the detail.

8 Conclusion

This paper studies technology choices and structural change in a multi-sector economy. Our key conclusion is that technology choices in each sector and structural change across industries are driven by changes in endowments. With an increase in capital endowment, industrial distribution moves towards capital intensive sectors, capital intensity of each industry becomes larger, and firms in each industry are using more capital/labor augmenting technology when the elasticity of substitution between capital and labor is larger/lower than one.

In this paper, we first build a multi-sector general equilibrium model to jointly discuss endowment structure, structural change and technology choices. Our model extends Caselli and Coleman (2006) to heterogeneous industries. The model shows that an increase in capital endowment drives up real wage, and further affects industrial structure and technology choices. We extend the methodology proposed by Caselli and Coleman (2006) to compute technology choices separately for panels of industries of the US. We find the data pattern is consistent with three propositions derived from the model. Firstly, we find US manufacturing industries use labor more efficiently, while Chinese manufacturing industries use capital more efficiently, with the increase in total capital endowment. Secondly, we find robust patterns of structural change in ISIC defined industries, from labor intensive to capital intensive sectors; thirdly, over time, capital intensity of each industry is increasing, while the relative order of capital intensity across industries remain stable.

We contribute to the literature by studying technology choices in multi-sector economy. Conventional wisdom believes that the economic growth in East Asia is not sustainable since it is mostly contributed by the capital accumulation rather than TFP growth, see Young (1995) and Krugman(1994). This paper provides an argument against it. We show that with the capital accumulation, there is continuous industrial upgrading towards the capital intensive sectors, accompanied by the adoption of new technologies. This results in high growth rate with low TFP. We also emphasize that the sequence of industrial upgrading and technology adoption depends on the endowment structure.

Our model predictions are consistent with empirical patterns. The model we build has elegant closed solutions, providing a workhorse for multi-sector structural analysis. The model can be used to analyze the sources of TFP growth, decomposing it into technological innovation (TFP) and changes in the technologies and industries, driven by accumulation of factors, rather than attributing all of it to

technological innovation. The model can also be used to study how financial structure should differ in countries with different levels of development and capital intensities in industries (Lin, Sun and Jiang 2013, Lin, Cull and Demirguc-Kunt, 2013). By incorporating various frictions and market failures into the model, we will be able to discuss different policy options and the role of state in the economy, for example by incorporating role of information, infrastructure and human capital in facilitating structural change from small-scale labor-intensive industries to large-scale capital-intensive industries. There will be a need for the government to provide information and the needed improvement in hard and soft infrastructures as well as human capital in the process of industrial upgrading. Further, the model can be used to explore the issues related to international trade and technology spill-overs across countries at different levels of development.

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Appendix I. Proofs

At first, we give two lemmas.

Lemma 1. For any $k_0 > 0$, there exists a unique z satisfying (2), and this z is strictly increasing with respect to k_0 .

Proof. In fact, it's easy to see that under the assumption **A3**, k_0 as a function of $z \in [0, \infty)$ is strictly increasing, taking values from 0 to ∞ , accordingly. And this completes the proof of the lemma.

Lemma 2. Assume that $0 < \underline{\alpha} < \bar{\alpha} < 1$, $a > 0$, $b > 0$ are fixed. Then, (α^*, K^*, L^*) satisfying

$$\alpha^* \in [\underline{\alpha}, \bar{\alpha}], \quad K^* > 0, \quad L^* > 0$$

is a solution of the optimization problem

$$\max_{\alpha, K, L} \{K^\alpha L^\beta - aK - bL\},$$

s.t.

$$\alpha \in [\underline{\alpha}, \bar{\alpha}], \quad \beta = 1 - \alpha, \quad K \geq 0, \quad L \geq 0,$$

if and only if

$$a = \alpha^* (k^*)^{-\beta^*}, \tag{12}$$

$$b = \beta^* (k^*)^{\alpha^*}, \tag{13}$$

where $\beta^* = 1 - \alpha^*$, and $k^* = K^*/L^*$, and

$$\alpha^* \in \arg \max_{\alpha \in \{\underline{\alpha}, \bar{\alpha}\}} \left\{ \beta \left(\frac{\alpha}{a} \right)^{\alpha/\beta} \right\}. \tag{14}$$

Proof. Necessity. Notice that

$$(K^*, L^*) \in \arg \max_{K, L} \{K^{\alpha^*} L^{\beta^*} - aK - bL\},$$

s.t.

$$K \geq 0, \quad L \geq 0,$$

it follows (12) and (13) immediately, which implies that

$$(K^*)^{\alpha^*} (L^*)^{\beta^*} - aK^* - bL^* = 0.$$

Noticing that for any $K \geq 0$, $L > 0$,

$$K^\alpha L^\beta - aK - bL = L(k^\alpha - ak - b),$$

where $k = K/L$, therefore,

$$(\alpha^*, k^*) \in \arg \max_{\alpha, k} (k^\alpha - ak),$$

s.t.

$$\alpha \in [\underline{\alpha}, \bar{\alpha}], \quad k \geq 0.$$

Let

$$F(\alpha, k) = k^\alpha - ak, \quad \alpha \in [\underline{\alpha}, \bar{\alpha}], \quad k \geq 0,$$

and

$$k(\alpha) =: \arg \max_{k \geq 0} F(\alpha, k).$$

One can easily get that

$$k(\alpha) = \left(\frac{\alpha}{a}\right)^{1/\beta},$$

and hence,

$$V(\alpha) =: F(\alpha, k(\alpha)) = \beta \left(\frac{\alpha}{a}\right)^{\alpha/\beta},$$

where $\beta = 1 - \alpha$. Therefore,

$$\alpha^* \in \arg \max_{\alpha} V(\alpha),$$

s.t.

$$\alpha \in [\underline{\alpha}, \bar{\alpha}].$$

Now, consider the function

$$G(\alpha) =: \ln V(\alpha) = \ln \beta + \frac{\alpha}{\beta} \ln \frac{\alpha}{a},$$

Since

$$\frac{dG}{d\alpha} = \frac{1}{\beta^2} \ln \frac{\alpha}{a},$$

and hence, function G is strictly decreasing for $\alpha < a$ and strictly increasing for $\alpha > a$. It follows that function G (and hence function V) takes its maximum only on the end points, that is, $\underline{\alpha}$ or $\bar{\alpha}$, it can not take its maximum at any interior points, and this gives (14).

Sufficiency. Obvious. The lemma is proved.

Lemma 3. Let $\alpha \in (0, 1)$, $\beta = 1 - \alpha$, $a > 0, b > 0$. Then,

(i)

$$\{(0, 0)\} = \arg \max_{K \geq 0, L \geq 0} (K^\alpha L^\beta - aK - bL)$$

if and only if

$$\left(\frac{\alpha}{a}\right)^\alpha \left(\frac{\beta}{b}\right)^\beta < 1;$$

(ii) there exists $K^* > 0, L^* > 0$ such that

$$(K^*, L^*) \in \arg \max_{K \geq 0, L \geq 0} (K^\alpha L^\beta - aK - bL)$$

if and only if

$$\left(\frac{\alpha}{a}\right)^\alpha \left(\frac{\beta}{b}\right)^\beta = 1.$$

Proof. Easy.

Now, we give the proof of Theorem 1.

Proof of Theorem 1. One can verify that the given solution is really an equilibrium, from which the sufficiency follows. In the sequel, we prove the necessity. Suppose that $(C_i; a_i, b_i, K_i, L_i; p_i, r, \omega)_{i \in [0,1]}$ is an equilibrium.

For any $i \in [0, 1]$, denote

$$r_i = \frac{r}{p_i}, \quad \omega_i = \frac{\omega}{p_i}, \quad k_i = \frac{K_i}{L_i}, \quad x_i = \left(\frac{a_i k_i}{b_i}\right)^{\rho_i}, \quad s_i = (\gamma_i x)^{\delta_i},$$

where

$$x =: \frac{\omega}{r}.$$

We first solve the individual's optimization problem. This is a typical isoperimetric problem. By the standard method of calculus of variation, we know that there exists a constant λ such that the optimal path of the consumption satisfies the Euler equation

$$\rho \theta_i C_i^{\rho-1} - \lambda p_i = F_{C_i} = \frac{d}{di} F_{\dot{C}_i} = 0, \quad \text{a.s.}$$

where $F = \theta_i C_i^\rho - \lambda p_i C_i$, and \dot{C}_i stands the derivative of C_i with respect to i , and hence, $\lambda > 0$, and

$$C_i^{1-\rho} = \mu \theta_i r_i, \tag{15}$$

where $\mu = 1/(\lambda r)$.

Now, we solve the firm's optimization problem. We know that in equilibrium, for any $i \in [0, 1]$, we have

$$0 = \max_{a_i, b_i, K_i, L_i, Y_i} \{Y_i - r_i K_i - \omega_i L_i\}, \tag{16}$$

s.t.

$$Y_i = ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i},$$

$$\left(\frac{a_i}{m_i}\right)^{\sigma_i} + \left(\frac{b_i}{n_i}\right)^{\sigma_i} = 1.$$

We first solve the embedded cost minimization problem:

$$Z_i =: \min_{K_i, L_i} \{r_i K_i + \omega_i L_i\}, \tag{17}$$

s.t.

$$Y_i = ((a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i})^{1/\rho_i},$$

the solution for which satisfies

$$r_i K_i = \mu_i \frac{(a_i K_i)^{\rho_i}}{(a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}} Y_i, \quad (18)$$

$$\omega_i L_i = \mu_i \frac{(b_i L_i)^{\rho_i}}{(a_i K_i)^{\rho_i} + (b_i L_i)^{\rho_i}} Y_i, \quad (19)$$

for some Lagrange multiplier $\mu_i > 0$, which depends only on a_i, b_i, Y_i , and hence,

$$Z_i = \mu_i Y_i. \quad (20)$$

From (18), (19), we get

$$\frac{r_i K_i}{\omega_i L_i} = \left(\frac{a_i K_i}{b_i L_i} \right)^{\rho_i} = \left(\frac{a_i / r_i}{b_i / \omega_i} \right)^{\tau_i},$$

therefore,

$$\begin{aligned} \omega_i L_i &= \mu_i b_i L_i \left(1 + \left(\frac{a_i K_i}{b_i L_i} \right)^{\rho_i} \right)^{1/\rho_i - 1} \\ &= \mu_i b_i L_i \left(1 + \left(\frac{a_i / r_i}{b_i / \omega_i} \right)^{\tau_i} \right)^{1/\tau_i}, \end{aligned}$$

which yields

$$\mu_i = \left(\left(\frac{a_i}{r_i} \right)^{\tau_i} + \left(\frac{b_i}{\omega_i} \right)^{\tau_i} \right)^{-1/\tau_i}. \quad (21)$$

From (16),(17),(20) and (21), we have

$$1 = \max_{a_i, b_i} \left(\left(\frac{a_i}{r_i} \right)^{\tau_i} + \left(\frac{b_i}{\omega_i} \right)^{\tau_i} \right)^{1/\tau_i},$$

s.t.

$$\left(\frac{a_i}{m_i} \right)^{\sigma_i} + \left(\frac{b_i}{n_i} \right)^{\sigma_i} = 1,$$

the solution for which satisfies

$$\left(\frac{a_i}{r_i} \right)^{\tau_i} = \left(\frac{a_i}{m_i} \right)^{\sigma_i} = \left(\frac{m_i}{r_i} \right)^{\delta_i} = \frac{s_i}{1 + s_i}, \quad (22)$$

$$\left(\frac{b_i}{\omega_i} \right)^{\tau_i} = \left(\frac{b_i}{n_i} \right)^{\sigma_i} = \left(\frac{n_i}{\omega_i} \right)^{\delta_i} = \frac{1}{1 + s_i}. \quad (23)$$

And hence, $\mu_i = 1$. Therefore, from (17),(20), and $Y_i = C_i$, we get

$$C_i = r_i K_i (1 + x_i^{-1}) = \omega_i L_i (1 + x_i). \quad (24)$$

It follows that for any $i \in [0, 1]$,

$$s_i = \left(\frac{a_i}{b_i \gamma_i} \right)^{\sigma_i} = \left(\frac{a_i}{b_i} x \right)^{\tau_i}, \quad \frac{k_i}{x_i} = x,$$

and hence,

$$x_i = \left(\frac{a_i}{b_i} k_i \right)^{\rho_i} = \left(\frac{a_i}{b_i} x x_i \right)^{\rho_i} = \left(\frac{a_i}{b_i} x \right)^{\tau_i} = s_i.$$

From (22),(23), we have

$$\begin{aligned} a_i &= m_i (1 + s_i^{-1})^{-1/\sigma_i}, \\ b_i &= n_i (1 + s_i)^{-1/\sigma_i}, \\ r_i &= m_i (1 + s_i^{-1})^{1/\delta_i}, \\ \omega_i &= n_i (1 + s_i)^{1/\delta_i}. \end{aligned} \tag{25}$$

From (15),(24), (25), we obtain

$$K_i^{1-\rho} = \mu \theta_i m_i^\rho (1 + x_i^{-1})^{\rho/\varepsilon_i - 1}.$$

Since

$$\int_0^1 K_i di = K_0,$$

then,

$$\mu = \frac{K_0}{\int_0^1 (\theta_i m_i^\rho (1 + x_i^{-1})^{\rho/\varepsilon_i - 1})^{1/(1-\rho)} di}.$$

Therefore,

$$K_i = \frac{(\theta_i m_i^\rho (1 + x_i^{-1})^{\rho/\varepsilon_i - 1})^{1/(1-\rho)}}{\int_0^1 (\theta_j m_j^\rho (1 + x_j^{-1})^{\rho/\varepsilon_j - 1})^{1/(1-\rho)} di} K_0, \tag{26}$$

Analogously,

$$L_i = \frac{(\theta_i n_i^\rho (1 + x_i)^{\rho/\varepsilon_i - 1})^{1/(1-\rho)}}{\int_0^1 (\theta_j n_j^\rho (1 + x_j)^{\rho/\varepsilon_j - 1})^{1/(1-\rho)} dj} L_0, \tag{27}$$

Dividing (26) by (27) in both sides simultaneously, and noticing $k_i = x x_i$ and $x_i = (\gamma_i x)^{\delta_i}$, we get

$$k_0 = \frac{x^{1/(1-\rho)} \int_0^1 [\theta_i m_i^\rho (1 + (\gamma_i x)^{-\delta_i})^{\rho/\varepsilon_i - 1}]^{1/(1-\rho)} di}{\int_0^1 [\theta_j n_j^\rho (1 + (\gamma_j x)^{\delta_j})^{\rho/\varepsilon_j - 1}]^{1/(1-\rho)} dj}.$$

And hence, the above x is just the z in (2). The theorem is proved.

The proof of Theorem 2 is similar to that of theorem 1, hence, omitted.

Proof of Theorem 3. First of all, one can easily verify that the given $(C_i^*; K_{ij}^*, L_{ij}^*; p_i^*, r^*, \omega^*)_{i=1,2,\dots,n,j=1,2}$ in the theorem is really an equilibrium. In the sequel, we prove the converse statement, that is, if $(C_i; K_{ij}, L_{ij}; p_i, r, \omega)_{i=1,\dots,n,j=1,2}$ is an equilibrium, then, it must satisfy all the conditions in the theorem.

We consider two cases.

First case. For this equilibrium, there exists some $t \in \{0, 1, \dots, n\}$ such that for any $i \leq t$, happens \bar{i} ; for any $i > t$, happens \underline{i} , there is no industry in which happens mixed technology.

By solving the individual's optimization problem, one can easily get that for any $i \in \{1, \dots, n\}$,

$$p_i C_i = \theta_i (r K_0 + \omega L_0). \quad (28)$$

By solving the firms' optimization problems, one can get that for any $i \in \{1, \dots, n\}$,

$$\frac{r}{p_i} = \alpha_i g_i^{-\beta_i}, \quad \frac{\omega}{p_i} = A_i \beta_i g_i^{\alpha_i}; \quad (29)$$

where

$$\begin{aligned} \alpha_i &= \bar{\alpha}_i, \quad i \leq t; \\ &= \underline{\alpha}_i, \quad i > t, \end{aligned}$$

and $\beta_i = 1 - \alpha_i$, $g_i = K_i / (A_i L_i)$, and

$$\begin{aligned} K_i &= K_{i2}, \quad L_i = L_{i2}, \quad i \leq t; \\ K_i &= K_{i1}, \quad L_i = L_{i1}, \quad i > t. \end{aligned}$$

By the market clearing condition, we have that for any $i \in \{1, \dots, n\}$,

$$C_i = K_i^{\alpha_i} (A_i L_i)^{\beta_i}, \quad (30)$$

and

$$\sum_{i=1}^n K_i = K_0, \quad \sum_{i=1}^n L_i = L_0. \quad (31)$$

From (28)(29)(30), we get that for any $i, j \in \{1, \dots, n\}$,

$$\begin{aligned} \frac{\theta_i}{\theta_j} &= \frac{p_i C_i}{p_j C_j} \\ &= \frac{\alpha_i^{-1} K_i}{\alpha_j^{-1} K_j} \\ &= \frac{\beta_i^{-1} L_i}{\beta_j^{-1} L_j}, \end{aligned}$$

which, combining with (31), gives that for any $i \in \{1, \dots, n\}$,

$$K_i = \frac{\theta_i \alpha_i}{\sum_{j=1}^n \theta_j \alpha_j} K_0, \quad L_i = \frac{\theta_i \beta_i}{\sum_{j=1}^n \theta_j \beta_j} L_0. \quad (32)$$

From (29)(32), we get that

$$\frac{\omega}{r} = \frac{\sum_{j \leq t} \theta_j \bar{\beta}_j + \sum_{j > t} \theta_j \underline{\beta}_j}{\sum_{j \leq t} \theta_j \bar{\alpha}_j + \sum_{j > t} \theta_j \underline{\alpha}_j} k_0, \quad (33)$$

In this case, since for any $i \leq t$, $K_{i1} = L_{i1} = 0$, $K_{i2} > 0$, $L_{i2} > 0$, and for any $i > t$, $K_{i1} > 0$, $L_{i1} > 0$, $K_{i2} = L_{i2} = 0$, by Lemma 3, we have that

$$\begin{aligned} \left(\frac{\alpha_i}{r}\right)^{\alpha_i} \left(\frac{\underline{\beta}_i}{\omega/A_i}\right)^{\underline{\beta}_i} &\leq \left(\frac{\bar{\alpha}_i}{r}\right)^{\bar{\alpha}_i} \left(\frac{\bar{\beta}_i}{\omega/A_i}\right)^{\bar{\beta}_i}, \quad \forall i \leq t; \\ \left(\frac{\alpha_i}{r}\right)^{\alpha_i} \left(\frac{\underline{\beta}_i}{\omega/A_i}\right)^{\underline{\beta}_i} &\geq \left(\frac{\bar{\alpha}_i}{r}\right)^{\bar{\alpha}_i} \left(\frac{\bar{\beta}_i}{\omega/A_i}\right)^{\bar{\beta}_i}, \quad \forall i > t. \end{aligned}$$

Therefore,

$$\sigma_t \leq \frac{\omega}{r} \leq \sigma_{t+1},$$

which, combining with (33), yields that

$$k_{2t}^* \leq k_0 \leq k_{2t+1}^*.$$

Second case. For this equilibrium, there exists some $t \in \{1, \dots, n\}$ such that for any $i < t$, happens \bar{i} ; for the $i = t$, happens i ; for any $i > t$, happens \underline{i} .

By solving the individual's optimization problem, (28) still holds. By solving the firms' optimization problem, we get that

$$\frac{r}{p_i} = \bar{\alpha}_i g_{i2}^{-\bar{\beta}_i}, \quad \frac{\omega}{p_i} = A_i \bar{\beta}_i g_{i2}^{\bar{\alpha}_i}, \quad i \leq t, \quad (34)$$

$$\frac{r}{p_i} = \underline{\alpha}_i g_{i2}^{-\underline{\beta}_i}, \quad \frac{\omega}{p_i} = A_i \underline{\beta}_i g_{i2}^{\underline{\alpha}_i}, \quad i \geq t. \quad (35)$$

And hence,

$$\begin{aligned} \frac{\omega}{r} &= \frac{\bar{\beta}_t}{\bar{\alpha}_t} k_{t2} = \frac{\underline{\beta}_t}{\underline{\alpha}_t} k_{t1}, \\ \bar{\beta}_t g_{t2}^{\bar{\alpha}_t} &= \underline{\beta}_t g_{t2}^{\underline{\alpha}_t}, \end{aligned}$$

which yields

$$\frac{\omega}{r} = \sigma_t.$$

Analogous to the above analysis, from (28)(34)(35), we get that for any $i \neq t$,

$$\begin{aligned} K_i &= \theta_i \alpha_i (K_0 + \sigma_t L_0), \\ L_i &= \theta_i \beta_i (K_0 / \sigma_t + L_0), \end{aligned}$$

where

$$\begin{aligned} K_i &= K_{i2}, \quad L_i = L_{i2}, \quad i < t; \\ K_i &= K_{i1}, \quad L_i = L_{i1}, \quad i > t, \end{aligned}$$

and

$$\begin{aligned} \frac{K_{t1}}{\underline{\alpha}_t} + \frac{K_{t2}}{\bar{\alpha}_t} &= \theta_t(K_0 + \sigma_t L_0), \\ \frac{L_{t1}}{\underline{\beta}_t} + \frac{L_{t2}}{\bar{\beta}_t} &= \theta_t(K_0/\sigma_t + L_0). \end{aligned}$$

By the market clearing condition, we get that

$$\begin{aligned} K_{t1} + K_{t2} &= K_0 - \sum_{i \neq t} \theta_i \alpha_i (K_0 + \sigma_t L_0); \\ L_{t1} + L_{t2} &= L_0 - \sum_{i \neq t} \theta_i \beta_i (K_0/\sigma_t + L_0). \end{aligned}$$

Therefore,

$$\begin{aligned} K_{t1} &= \frac{\sum_{j \leq t} \theta_j \bar{\beta}_j + \sum_{j > t} \theta_j \underline{\beta}_j}{\bar{\alpha}_t/\underline{\alpha}_t - 1} \left(\frac{k_{2t}^*}{k_0} - 1 \right) K_0, \\ K_{t2} &= \frac{\sum_{j < t} \theta_j \bar{\beta}_j + \sum_{j \geq t} \theta_j \underline{\beta}_j}{1 - \underline{\alpha}_t/\bar{\alpha}_t} \left(1 - \frac{k_{2t-1}^*}{k_0} \right) K_0; \\ L_{t1} &= \frac{\sum_{j \leq t} \theta_j \bar{\alpha}_j + \sum_{j > t} \theta_j \underline{\alpha}_j}{1 - \bar{\beta}_t/\underline{\beta}_t} \left(1 - \frac{k_0}{k_{2t}^*} \right) L_0, \\ L_{t2} &= \frac{\sum_{j < t} \theta_j \bar{\alpha}_j + \sum_{j \geq t} \theta_j \underline{\alpha}_j}{\underline{\beta}_t/\bar{\beta}_t - 1} \left(\frac{k_0}{k_{2t-1}^*} - 1 \right) L_0. \end{aligned}$$

And, obviously, in this case,

$$k_{2t-1}^* \leq k_0 \leq k_{2t}^*.$$

Now, by the same method, one can prove that any other cases are impossible. And this completes the proof of the theorem.

The proofs of Theorem 4 and Corollary 7 are easy, hence, omitted.

Appendix II. Supplementary Figures

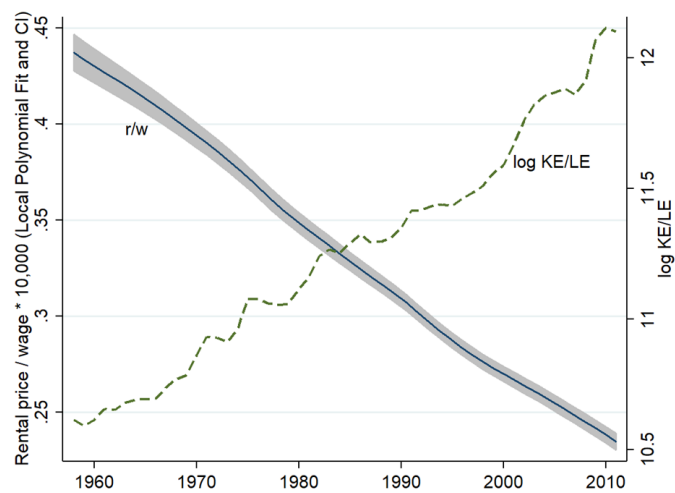


Figure A1: Factor Returns in US