



No. E2019008

2019-10-27

## China's Housing Bubble, Infrastructure Investment, and Economic Growth

Shenzhe Jiang, Jianjun Miao, and Yuzhe Zhang\*

October 26, 2019

### Abstract

China's housing prices have been growing rapidly over the past decades, despite the low rents growth. We study the impact of housing bubbles on China's real economy, through the channel that local governments use land-sale revenues to fuel infrastructure investment. We calibrate our model to the Chinese data over the period 2003-2013 and find that our calibrated model can match the declining capital return and GDP growth, the average housing price growth, and the rising infrastructure to GDP ratio in the data. We conduct two counterfactual experiments. If the bubble bursts in 2025, then on impact the GDP growth rate falls to 2.6% due to the hit to the housing sector, but GDP in the long run exceeds that with the bubble because of the reallocation of capital and labor to the nonhousing sector. If the bubble remains, however, implementing a property tax and using the tax revenue to finance infrastructure investment will reduce the housing price and increase long-run GDP.

**Keywords:** Housing Bubble, Infrastructure, Economic Growth, Chinese Economy, Property Tax.

**JEL codes:** O11, O16, O18, P24, R21, R31.

*The series of New Structural Economics Working Papers aims to encourage academic scholars and students from all over the world to conduct academic research in the field of new structural economics. Excellent papers are selected irregularly and are offered academic suggestions and recommendation, but the published working papers are not intended to represent official communication from INSE. The Present Paper was approved by the NSE A0 Growth Group.*

---

\*Jiang: Institute of New Structural Economics, Peking University, Beijing, China. Email: shenzhejiang@nsd.pku.edu.cn. Miao: Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA. Email: miaoj@bu.edu. Zhang: Department of Economics, Texas A&M University, College Station, TX, 77843. Email: zhangeager@tamu.edu. We thank Kaiji Chen, Li Gan, Dennis Jansen, Guoqiang Tian, Anastasia Zervou, Sarah Zubairy, Pedro Bento, and seminar participants at Texas A&M University for helpful comments.

# China's Housing Bubble, Infrastructure Investment, and Economic Growth

Shenzhe Jiang, Jianjun Miao, and Yuzhe Zhang\*

October 26, 2019

## Abstract

China's housing prices have been growing rapidly over the past decades, despite the low rents growth. We study the impact of housing bubbles on China's real economy, through the channel that local governments use land-sale revenues to fuel infrastructure investment. We calibrate our model to the Chinese data over the period 2003-2013 and find that our calibrated model can match the declining capital return and GDP growth, the average housing price growth, and the rising infrastructure to GDP ratio in the data. We conduct two counterfactual experiments. If the bubble bursts in 2025, then on impact the GDP growth rate falls to 2.6% due to the hit to the housing sector, but GDP in the long run exceeds that with the bubble because of the reallocation of capital and labor to the nonhousing sector. If the bubble remains, however, implementing a property tax and using the tax revenue to finance infrastructure investment will reduce the housing price and increase long-run GDP.

**Keywords:** Housing Bubble, Infrastructure, Economic Growth, Chinese Economy, Property Tax.

**JEL codes:** O11, O16, O18, P24, R21, R31.

---

\*Jiang: Institute of New Structural Economics, Peking University, Beijing, China. Email: shenzhe-jiang@nsd.pku.edu.cn. Miao: Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA. Email: miaoj@bu.edu. Zhang: Department of Economics, Texas A&M University, College Station, TX, 77843. Email: zhangeager@tamu.edu. We thank Kaiji Chen, Li Gan, Dennis Jansen, Guoqiang Tian, Anastasia Zervou, Sarah Zubairy, Pedro Bento, and seminar participants at Texas A&M University for helpful comments.

# 1 Introduction

China implemented a series of market-oriented housing reforms in the 1990s. Since then, the Chinese real estate market has experienced a dramatic and long-lasting boom. This boom has an important impact on the Chinese macroeconomy. Based on the annual data during the period 2003-2013, we find the following stylized facts as detailed in Section 2:

- The growth rates of GDP were high on average (10%) and declined over time.
- The growth rates of housing prices were high on average (10%) and the growth rates of rents were low on average (0.5%).
- The residential investment to GDP ratio was high on average (8.6%) and increased over time.<sup>1</sup>
- The government land-sale revenue to GDP ratios increased over time.
- The infrastructure investment to GDP ratios increased over time.<sup>2</sup>
- The returns to capital were high on average (10%) and declined over time.

In this paper we propose a two-sector overlapping-generations (OLG) model to explain these facts. The model features a housing sector that produces houses using land, capital, and labor as inputs, and a nonhousing sector that produces a final nonhousing good using capital and labor as inputs. There are two key ingredients of our model. First, rational expectations of lower returns to capital in the long run can induce current generations of entrepreneurs to seek alternative stores of value for their rapidly growing wealth. In a financially underdeveloped economy with a shortage of financial assets, housing becomes a natural investment option for current entrepreneurs, who rationally anticipate a strong demand for such an asset by future generations. Speculation and low growth rates of housing rents together sustain a self-fulfilling growing housing bubble.

Second, we incorporate China's institutional feature of land policies. In China land is state owned and local governments collect land-transferring fees through land sales. High land prices associated with high housing prices generate a large revenue for local governments, and the Chinese law requires that a certain fraction of the land-sale revenue be used toward infrastructure investment. Thus a housing bubble can lead to an increased infrastructure investment, which in turn facilitates production and raises nonhousing firms' productivity. This crowding-in effect of a housing bubble can raise GDP and economic growth.

---

<sup>1</sup>In contrast, the US average ratio was 4.2% and the highest was 6.7% according to the US quarterly data over 2003Q1-2013Q4.

<sup>2</sup>Based on the IMF Investment and Capital Stock data set, China had the highest ratio of the infrastructure stock to the capital stock among the 15 largest economies in 2015.

On the other hand, a housing bubble can harm economic growth, because of the resource reallocation effect and the traditional crowding-out effect on capital accumulation (Tirole (1985)). In particular, purchases of the housing asset crowd out entrepreneurs' resources used to make capital investment. Moreover, in our two-sector model, capital and labor flow into the housing sector from the nonhousing sector in the wake of rising housing prices. Since the housing sector accounts for a relatively small share of the aggregate economy, this resource reallocation effect causes GDP to decline.

After using a simplified model to illustrate our story in Section 3, we calibrate an extended model to confront the Chinese data during the period 2003-2013. We find that our quantitative model can explain the stylized facts described above. We also conduct a growth accounting exercise to understand how housing bubbles affect economic growth. We find that the decline of the GDP growth over 2003-2013 is due to the decline of the nonhousing sector growth, which is driven mainly by the decline of the capital growth, as capital flows from the nonhousing sector into the bubbly housing sector.

A standard model without housing bubble has difficulty in explaining the above stylized facts. Such a model implies that the housing price and rents grow at the same rate in the long run and cannot generate a long-lasting boom of housing prices given the very low rent growth in China. As a result, the standard model also has difficulty in explaining the rapid growth of land-sale revenue, residential investment, and infrastructure investment.

We next consider two counterfactual experiments. There have been substantial concerns in China's academic and policy circles that the rising housing prices might have developed into a gigantic housing bubble, which might eventually burst and damage China's economy. To control housing prices, the Chinese government has considered a property tax for more than a decade, but has not implemented it so far. We use our calibrated model to answer two counterfactual questions: (1) what is the consequence of a housing bubble crash? and (2) how does the adoption of the property tax affect the economy?

For the first question, we suppose that the housing bubble collapses in 2025, and then simulate the equilibrium dynamics afterwards. Unsurprisingly, the market prices of all existing houses take a big hit. Since newly built houses enter GDP, China's GDP growth rate decreases from 5% to 2.6% in 2025. After a few years, GDP gradually recovers and rises above the level in the case where the bubble never collapses. The reason is that the collapsed housing price reduces the government's land-sale revenue, and consequently fewer resources are invested in infrastructure and housing assets. With more capital invested in the nonhousing sector and with labor flowing back into the nonhousing sector, increased output from this sector makes up for the lost value of new houses.

To answer the second question, we suppose that the Chinese government imposes a permanent 1.5% property tax on all homes starting from 2025 and uses the property tax revenue to finance

infrastructure investment.<sup>3</sup> This policy will immediately reduce the bubble size in 2025 because the after-tax return of owning a home is lower. GDP also declines in 2025. However, after 30 years, GDP is 18.5% higher due to higher accumulation of capital as entrepreneurs invest more in capital and capital also flows from the housing sector into the nonhousing sector.

**Related literature.** Our paper is related to three strands of the literature and contributes to the literature by providing the first quantitative study of the impact of the Chinese housing bubbles on infrastructure investment and economic growth.

First, our paper is related to the recent literature on housing bubbles (Arce and López-Salido (2011), Zhao (2015), Chen and Wen (2017), and Dong et al. (2019)).<sup>4</sup> Arce and López-Salido (2011) and Zhao (2015) consider endowment economies in the OLG framework, while Dong et al. (2019) introduce production in an infinite-horizon growth model. Our paper is most closely related to Chen and Wen (2017) with three main differences. First, Chen and Wen (2017) do not distinguish between land and housing and assume that they are a pure bubble asset without fundamentals. In our model, housing firms use land as an input to produce houses, which pay rents so that houses have fundamental values. Second, Chen and Wen (2017) only focus on the crowding-out effect of the housing bubble as in Tirole (1985).<sup>5</sup> They do not study the Chinese institutional feature that the government uses land sales to finance infrastructure investment and the associated crowding-in effect. Finally, Chen and Wen (2017) assume that housing supply is exogenously fixed, while we explicitly model the endogenous supply in the housing sector as in Dong et al. (2019). Chen and Wen (2017) consider two sectors with state-owned and private firms producing the same final good, while we study two sectors with housing and nonhousing firms producing different products.

Our paper is also related to the second strand of the literature that studies how asset bubbles crowd in capital through the collateral channel as in Kiyotaki and Moore (1997), including Martin and Ventura (2012), Farhi and Tirole (2012), Miao et al. (2015), Hirano and Yanagawa (2016), and Miao and Wang (2014, 2018). In these papers bubbly assets can be used as collateral or simply raise net worth. The burst of a bubble tightens the firms' credit constraints, forcing them to cut back investment. Although the collateral channel of the housing bubble is essential in understanding the impact of the crashes in the 1989 Japanese housing market and in the 2007 U.S. housing market, its importance in China is still under debate.

In an empirical study, Wu et al. (2015) find that the collateral channel effect in China does not exist either for firms overall or for private firms, while Chen et al. (2015) provide empirical evidence

---

<sup>3</sup>The average property tax rate in the US across all states is around 1.4%.

<sup>4</sup>There is also a literature that studies housing prices without bubbles using DSGE models. Important papers include Iacoviello (2005), Iacoviello and Neri (2010), and Liu et al. (2013), among others. Our paper does not follow this approach.

<sup>5</sup>In an empirical study, Chen et al. (2017) find evidence that a higher land price crowds out firms' investment unrelated to acquiring commercial land.

that this effect is significant for private firms, but not significant for state-owned enterprises. As Song et al. (2011) point out, most of private firms' investment comes from self-financing and only 10% comes from bank loans. Since private firms' investment accounted for around 25% of total investment during 2006-2013, even if we assume that all loans to private firms require residential housing as collateral, only 2.5% of total investment would be affected by the real estate price through the collateral channel. This number can be smaller in the data because it is typically small private firms that use housing as collateral. For all these reasons, neither Chen and Wen (2017) nor our model considers the collateral channel of housing prices.

Miao and Wang (2014) study a two-sector infinite-horizon model of stock price bubbles based on the collateral channel. They show that the emergence of a bubble in one sector may misallocate resources and retard economic growth. They do not focus on housing bubbles and their quantitative implications as in this paper.

Finally, our paper is related to a large literature on the role of government spending in economic growth, e.g., Barro (1990), Baxter and King (1993), Glomm and Ravikumar (1994), and Bassetto and Sargent (2006). As in the literature, efficiency in our model requires a good balance between infrastructure investment and private capital investment. This literature does not study housing bubbles and their impact on infrastructure investment. Our model also differs from this literature in modeling the government budget constraint. While infrastructure is purely funded by tax revenues in the literature, here it is also funded by the government sale of land to the housing sector.

Xiong (2019) uses a tournament model to show that Chinese local governments have a strong incentive to invest in infrastructure. He does not study our central issue of housing bubbles. We try to uncover local governments' source of funding. In particular, we emphasize the channel of the land-sale revenue, which accounts for more than half of local governments' revenue and is the collateral for more than half of their debt. Supporting our modeling, Mo (2018) finds evidence that Chinese local governments tend to increase investment in infrastructure when holding a large share of land-sale revenue in the total government revenue.

## 2 Stylized Facts

In this section we first describe some stylized facts based on China's aggregate annual data over 2003-2013 and then provide empirical evidence that supports our model mechanism based on China's province-level data.

### 2.1 Aggregate Evidence

As Wu et al. (2014), Chen and Wen (2017), and Fang et al. (2015) point out, the official national housing price indices published by the Chinese government tend to underestimate housing price growth due to measurement problems and the failure of controlling for housing quality. To correct

these issues, Wu et al. (2014) and Fang et al. (2015) propose new methods to construct Chinese housing price indices. We adopt the data of Fang et al. (2015) because they cover 120 major cities over 2003-2013, while the data of Wu et al. (2014) cover only 35 major cities over 2006-2010. As a result, we focus all our (annual) data on the period over 2003-2013 and find the following stylized facts (see Figure 1):

- High and declining GDP growth. The average growth rate of GDP was 10% based on the data from the China Statistical Yearbook. The GDP growth rate decreased from more than 14% to 7% during 2003-2013.
- High growth rates of housing prices and low growth rates of rents. After adjusting for inflation, we find that the average growth rate of real housing prices was 10%. By contrast, the average growth rate of housing rents was only 0.5%. The housing rents correspond to the urban household renting price indices taken from the National Bureau of Statistics of China (NBSC).
- Increasing residential investment to GDP ratios. The residential investment to GDP ratio increased from 6% in 2003 to 11% in 2013 based on the data from the NBSC. The average ratio during this period was 8.6%.
- Increasing land-sale revenue to GDP ratios. The land-sale revenue data are taken from the Finance Yearbook of China issued by the Chinese Ministry of Finance. The land-sale revenue increased from 4% of GDP to 7% of GDP, and became the most important source of income for Chinese local governments. The average ratio during 2003-2013 was 4.9%. The land-sale revenue accounted for 25% of total fiscal income on average over 2003-2013 and the share increased to more than 30% after 2009.
- Increasing infrastructure investment to GDP ratios. China does not directly report the public investment data. Following Jin (2016) and Wu et al. (2019), we define infrastructure investment as the total investment across four industries: (1) production and supply of electricity, gas, and water; (2) transport, storage, and post; (3) information transmission, computer services, and software (or telecommunications and other information transmission services); and (4) management of water conservancy, environment and public facilities. The infrastructure investment to GDP ratio increased dramatically from 6.5% to 9.7% during 2003-2013 based on the data from the NBSC. The average ratio during this period was 7.5%.
- High average and declining capital returns. Bai et al. (2006) construct the post-tax capital return data during 1978-2005 and Bai and Zhang (2014) extend these data to 2013. Based on their data, we find that the average capital return was 10% during 2003-2013. The capital return dropped dramatically from 15% in 2003 to 5% in 2013.

[Insert Figure 1 Here.]

## 2.2 Cross-province Evidence

In this subsection we provide micro-level evidence to support our key model mechanism: high housing price growth stimulates infrastructure investment, but crowds out capital investment and labor in the nonhousing sector.

We adopt annual province-level data from the NBSC, which reports annual GDP, fixed asset investment in different sectors, and average newly built housing prices for 31 provinces. Infrastructure investment is measured by the province-level fixed asset investment in infrastructure. Capital investment is computed as the gross fixed asset investment minus infrastructure and residential investments. Housing prices are deflated by the CPI, all investment data are deflated by the investment goods price index, and GDP is deflated by the GDP deflator. We use the population working in the manufacturing sector as a proxy for the employment (labor) in the nonhousing sector. The labor data cover 2008-2015 and all other data cover 2003-2015.

The regressions are specified as follows:

$$y_{i,t} = \alpha_i + \gamma_t + \beta_0 * growth\_hp_{i,t} + \beta_1 * growth\_gdp_{i,t} + \epsilon_{i,t},$$

where  $growth\_hp_{i,t}$  is the growth rate of housing prices in province  $i$  at year  $t$ ,  $growth\_gdp_{i,t}$  is the GDP growth rate in province  $i$  in year  $t$ , and  $\alpha_i$  and  $\gamma_t$  are province fixed effect and year fixed effect. The variable  $y_{i,t}$  represents the growth rates of infrastructure investment ( $growth\_infr$ ), capital investment ( $growth\_capital$ ), and labor ( $growth\_labor$ ), respectively. The standard errors are dually clustered by province and year. The regression results are reported in Table 1.

We find that all slope coefficients of housing price growth are significant. The coefficient is positive for infrastructure investment, but negative for capital investment and labor. These results show that high price growth is associated with increased infrastructure investment growth, but decreased capital investment growth and decreased labor growth in the nonhousing sector.

## 3 Basic Model

In this section we provide a small open economy two-sector OLG model of housing bubbles based on Tirole (1985) and Chen and Wen (2017). We make the following simplifying assumptions to illustrate the main model mechanism: (1) there is no population growth or technical progress; (2) the housing asset does not pay any rents and therefore is a pure bubble, (3) capital depreciates fully, and (4) the government runs a balanced budget. We will relax these assumptions in Section 4 to confront the data.



Table 1: Cross-province panel regressions

VARIABLES	(1)	(2)	(3)
	<i>growth_infr</i>	<i>growth_capital</i>	<i>growth_labor</i>
<i>growth_hp</i>	0.0645*	-0.1832***	-0.0969**
	(0.032)	(0.043)	(0.037)
<i>growth_gdp</i>	0.3278	1.2913***	0.1101
	(0.288)	(0.183)	(0.195)
Observations	372	372	217
Adjusted R-squared	0.284	0.410	0.232
Province	Yes	Yes	Yes
Year	Yes	Yes	Yes

Note: Robust standard errors in parentheses \*\*\* $p < 0.01$ , \*\* $p < 0.05$ , \* $p < 0.1$ .

### 3.1 Households

As in Song et al. (2011) and Chen and Wen (2017), there are two types of households in our small open economy: workers and entrepreneurs. They both live for two periods. Time runs forever and is denoted by  $t = 0, 1, 2, \dots$ . At the initial time  $t = 0$ , there is an old worker who is endowed with bonds  $b_0$ , and there is an old entrepreneur who is endowed with  $k_0$  units of capital and  $h_0$  units of a housing asset. In each period  $t \geq 0$ , a young worker and a young entrepreneur are born to replace the old. Each young worker supplies one unit of labor inelastically. After receiving their wage income, young workers choose consumption and savings. Because they are assumed to be out of the domestic capital and housing markets, they save only through risk-free bonds. The optimization problem for a newborn worker of age 1 is given by

$$\begin{aligned}
 \max \quad & \log(c_{1,t}^w) + \beta \log(c_{2,t+1}^w) \\
 \text{s.t.} \quad & c_{1,t}^w + b_{t+1} = w_t, \\
 & c_{2,t+1}^w = R^f b_{t+1},
 \end{aligned}$$

where  $c_{1,t}^w$  and  $c_{2,t+1}^w$  are his/her consumption when young and old,  $\beta \in (0, 1)$  is the discount factor,  $w_t$  is the wage rate,  $b_{t+1}$  is the holding of the risk-free bond, and  $R^f$  is the exogenous interest rate in the international financial market.

Entrepreneurs have the same preferences as workers. After inheriting an initial wealth level  $m_t$  from an old entrepreneur, a young entrepreneur of age 1 in period  $t$  can invest in both capital and housing to solve the following problem:

$$\begin{aligned}
 \max \quad & \log(c_{1,t}^e) + \beta \log(c_{2,t+1}^e) \\
 \text{s.t.} \quad & c_{1,t}^e + k_{t+1} + Q_t h_{t+1} = m_t, \\
 & c_{2,t+1}^e = R_{t+1} k_{t+1} + Q_{t+1} (1 - \delta_h) h_{t+1},
 \end{aligned}$$

where  $c_{1,t}^e$  and  $c_{2,t+1}^e$  are his consumption when young and old,  $h_{t+1} \geq 0$  is his holdings of the housing asset,  $Q_t \geq 0$  is the price of housing,  $k_{t+1} \geq 0$  is his holdings of capital,  $R_{t+1}$  is the capital return between periods  $t$  and  $t+1$ , and  $\delta_h$  is the depreciation rate of housing. In the simple model housing is a pure bubble asset without any fundamentals and hence its fundamental value is zero. Entrepreneurs trade houses for speculation. Assume that entrepreneurs cannot borrow and  $R_f < R_{t+1}$ .<sup>6</sup> Then entrepreneurs will not hold any bond in equilibrium. We will relax the no-borrowing assumption in Section 4.

Since the utility function is logarithmic, the entrepreneur's optimal saving is given by

$$k_{t+1} + Q_t h_{t+1} = \frac{\beta}{1 + \beta} m_t. \quad (1)$$

He/she will invest in both capital and housing only if the following no-arbitrage condition is satisfied:

$$R_{t+1} = \frac{Q_{t+1}(1 - \delta_h)}{Q_t} \text{ for } Q_t > 0. \quad (2)$$

That is, the returns on housing and capital are the same.

### 3.2 Nonhousing Sector

Each old entrepreneur owns a firm that produces the final consumption good using capital and labor as inputs. After investment of  $k_{t+1}$  at time  $t$ , each old entrepreneur at  $t+1$  receives output given by

$$y_{t+1} \equiv \hat{A}_{t+1}^\theta k_{t+1}^\alpha n_{c,t+1}^{1-\alpha}, \quad \alpha \in (0, 1)$$

where  $k_{t+1}$  and  $n_{c,t+1}$  are the firm's capital and labor,  $\hat{A}_{t+1}$  is the firm's productivity, and  $\theta > 0$  is an elasticity parameter. Following Glomm and Ravikumar (1994), we assume that the firm's productivity depends on infrastructure in the following way:

$$\hat{A}_{t+1} \equiv A_{t+1} / (K_{t+1}^\rho N_{c,t+1}^{1-\rho}),$$

where  $A_{t+1}$  is the aggregate infrastructure stock and  $\rho \in (0, 1)$  is a parameter. We normalize  $A_{t+1}$  by aggregate capital  $K_{t+1}$  and aggregate labor  $N_{c,t+1}$  for two reasons. First, in many cases, such as highways, utilities, and bridges, the productivity of infrastructure is indeed diluted by congestion when more people or firms use the same piece of infrastructure. Second, as Glomm and Ravikumar (1994) point out, in a model with endogenous infrastructure, the steady-state or balanced-growth path is not guaranteed if  $A_{t+1}$  is not normalized.

The old entrepreneur in period  $t+1$  pays the government output tax at the rate  $\tau$ , pays the worker  $w_{t+1} n_{c,t+1}$  as wage, and pays the young entrepreneur a fraction  $\psi$  of after-wage income  $(1 - \tau)y_{t+1} - w_{t+1} n_{c,t+1}$  as his initial wealth, i.e.,

$$m_{t+1} = \psi ((1 - \tau)y_{t+1} - w_{t+1} n_{c,t+1}). \quad (3)$$

---

<sup>6</sup>We will verify this assumption in equilibrium. During 2003-2013, China's average capital return is above 10 percent while its deposit rate is around zero.

The remainder is  $R_{t+1}k_{t+1}$ , which can be written as

$$R_{t+1}k_{t+1} \equiv \max_{n_{c,t+1}} (1 - \psi) \left[ (1 - \tau) \hat{A}_{t+1}^\theta k_{t+1}^\alpha n_{c,t+1}^{1-\alpha} - w_{t+1} n_{c,t+1} \right].$$

We make the following assumption on parameters:

**Assumption 1**  $\alpha - \rho\theta > 0$  and  $\alpha + (1 - \rho)\theta < 1$ .

The first inequality in this assumption guarantees the marginal return to capital is positive. The second inequality guarantees the return on the whole reproducible part (infrastructure and capital) is weakly decreasing.

Let

$$\phi_t \equiv \frac{Q_t h_{t+1}}{k_{t+1} + Q_t h_{t+1}}$$

denote the fraction of housing investment in a young entrepreneur's saving. By (1) and the definition of  $m_t$ , we can derive

$$k_{t+1} = (1 - \phi_t) \frac{\beta}{1 + \beta} m_t = (1 - \phi_t) \frac{\beta}{1 + \beta} \psi \alpha (1 - \tau) y_t. \quad (4)$$

Clearly, the emergence of a housing bubble in the sense that  $Q_t > 0$  crowds out capital investment as  $\phi_t \in (0, 1)$ .

### 3.3 Housing Sector

There is a continuum of competitive firms producing houses using labor and land as inputs. We do not consider capital input in the basic model for simplicity. We will relax this assumption in our quantitative model of Section 4. Each housing firm purchases land from the government that is the sole supplier at the price  $p_{Lt}$  at time  $t$ . Workers are freely mobile across the housing and nonhousing sectors. Each housing firm sells newly built houses to entrepreneurs at the price  $Q_t$ . Its profit maximization problem is given by

$$\max_{l_t, n_{h,t}} Q_t l_t^{\alpha_l} n_{h,t}^{1-\alpha_l} - p_{Lt} l_t - w_t n_{h,t},$$

where  $\alpha_l \in (0, 1)$  is an elasticity parameter, and  $l_t$  and  $n_{h,t}$  are the demand for land and labor, respectively.

Due to the constant-returns-to-scale technology, aggregation implies that total newly built houses  $Y_{h,t}$  satisfy

$$Y_{h,t} = L_t^{\alpha_l} N_{h,t}^{1-\alpha_l},$$

where  $L_t$  is the aggregate land supply set exogenously by the government and  $N_{h,t}$  is the aggregate labor hired by the housing sector. The total housing stock  $H_t$  evolves according to

$$H_{t+1} = (1 - \delta_h) H_t + Y_{h,t}.$$

### 3.4 Government and Infrastructure

The government supplies  $L_t$  units of land to the market exogenously in period  $t$ . To guarantee the existence of a bubbly steady state, we assume that  $\lim_{t \rightarrow \infty} L_t = L^* > 0$ , where  $L^*$  is the land supply in the long run.

For simplicity suppose that the government runs a balanced budget without issuing bonds. Its only spending is the infrastructure investment. Thus the government infrastructure expenditure is equal to its total revenue  $\tau Y_t + p_{Lt} L_t$ , where  $Y_t$  denotes aggregate final good output. The stock of infrastructure evolves as

$$A_{t+1} = (1 - \delta_a)A_t + \tau Y_t + p_{Lt} L_t, \quad (5)$$

where  $\delta_a$  is the depreciation rate of infrastructure.

### 3.5 Resource Constraint

The budget constraints of workers imply

$$c_{1,t}^w + c_{2,t}^w + b_{t+1} = w_t + R^f b_t, \quad (6)$$

where  $c_{1,t}^w + c_{2,t}^w$  is the sum of the consumption of old workers of generation- $(t-1)$  and young workers of generation- $t$ . In the domestic market the resource constraint is

$$A_{t+1} - (1 - \delta_a)A_t + c_{1,t}^e + c_{2,t}^e + K_{t+1} + w_t = Y_t. \quad (7)$$

Because of constant-returns-to-scale technology, aggregate nonhousing output satisfies

$$Y_t = \hat{A}_t^\theta K_t^\alpha N_{c,t}^{1-\alpha},$$

where  $N_{c,t} = 1 - N_{h,t}$  denotes aggregate labor in the nonhousing sector. Notice that  $Y_t$  does not measure GDP, which also includes housing production.

Summing up (6) and (7) yields

$$A_{t+1} - (1 - \delta_a)A_t + K_{t+1} + c_{1,t}^e + c_{2,t}^e + c_{1,t}^w + c_{2,t}^w + b_{t+1} - R^f b_t = Y_t. \quad (8)$$

On the left side of equation (8),  $A_{t+1} - (1 - \delta_a)A_t$  is the infrastructure investment,  $K_{t+1}$  is the private capital investment,  $b_{t+1} - R^f b_t$  is the surplus in the current account, and the rest is aggregate consumption.

### 3.6 Equilibrium

The equilibrium of the economy is defined as follows.

**Definition 1** An equilibrium is a sequence of prices  $\{w_t, R_t, Q_t, p_{Lt}\}_{t=0}^{\infty}$ , savings  $\{b_t, k_t, h_t\}_{t=0}^{\infty}$ , consumption  $\{c_{1,t}^w, c_{2,t+1}^w, c_{1,t}^e, c_{2,t+1}^e\}_{t=0}^{\infty}$ , labor supply/demand  $\{N_{c,t}, N_{h,t}\}_{t=0}^{\infty}$ , and infrastructure  $\{A_t\}_{t=0}^{\infty}$  such that (i) workers and entrepreneurs maximize their life-time utilities; (ii) firms maximize profits; (iii) the government budget constraint (5) is satisfied; and (iv) the labor, capital, land, and housing markets clear.

In the rest of this paper, we call an equilibrium *bubbleless* if  $Q_t = 0$  for all  $t$ , and call an equilibrium *bubbly* if  $Q_t > 0$  for all  $t$ . While the former equilibrium always exists, the latter depends on parameters. In the bubbleless equilibrium, the housing price is zero and hence the land price  $p_{Lt}$  is also zero. Thus the housing and land markets disappear. The government finances infrastructure investment using output taxes only.

There are two steady states in our basic model: one is bubbleless and the other is bubbly. We use a variable without a time subscript to denote its steady-state value and add superscript  $n$  or  $b$  to denote its bubbleless or bubbly steady-state value, respectively. We first consider the bubbleless steady state and show that

$$A^n = \delta_a^{-1} \tau Y^n, \quad (9)$$

$$K^n = \frac{\beta}{1 + \beta} \alpha \psi (1 - \tau) Y^n, \quad (10)$$

where (9) follows from (5) with  $p_L^n = 0$ , and (10) follows from (4) with  $\phi^n = 0$ . Moreover, the bubbleless steady-state return on capital is equal to

$$R^n = \frac{(1 - \psi)(1 - \tau) \alpha Y^n}{K^n} = \frac{(1 - \psi)(1 + \beta)}{\psi \beta}. \quad (11)$$

Next we analyze the bubbly steady state in the following proposition. Its proof and the proofs of other results in the paper are relegated to Appendix A.

**Proposition 1** A unique bubbly steady state exists if

$$z \equiv \frac{(1 - \psi)(1 + \beta)}{(1 - \delta_h) \psi \beta} < 1. \quad (12)$$

Moreover,

$$\phi^b = 1 - z, \quad (13)$$

$$N_c^b = \frac{(1 - \phi^b)(1 - \alpha)(1 - \delta_h)}{(1 - \phi^b)(1 - \alpha)(1 - \delta_h) + \phi^b \delta_h (1 - \alpha) \alpha (1 - \psi)}, \quad (14)$$

$$A^b = \delta_a^{-1} \alpha_l \delta_h \phi^b \frac{\beta}{1 + \beta} \alpha \psi (1 - \tau) Y^b + \delta_a^{-1} \tau Y^b, \quad (15)$$

$$K^b = (1 - \phi^b) \frac{\beta}{1 + \beta} \alpha \psi (1 - \tau) Y^b. \quad (16)$$

The variable  $z$  defined in (12) is equal to the capital share in the entrepreneur's saving by (13). There are two senses in which condition  $z < 1$  is needed for the existence of a bubbly steady state. First, the bubble accounts for a fraction  $\phi^b = 1 - z$  of the entrepreneur's saving. We need  $z < 1$  to guarantee  $\phi^b > 0$ . Second, it follows from (11) that condition (12) requires that  $R^n < 1 - \delta_h = R^b$ . Imposing an upper bound on the bubbleless steady-state capital return is a standard assumption in the literature for a bubble to exist. As pointed out by Tirole (1985), only if the bubbleless steady-state capital return is sufficiently low, a bubbly asset can be traded as an alternative channel to save. Our upper bound  $1 - \delta_h$  is less than the standard value of 1 from the literature on bubbles, because the bubbly steady-state return on housing (which is also equal to the capital return) is  $1 - \delta_h$  due to housing depreciation by (2).

Equations (15) and (16) show that the bubbly steady-state levels of infrastructure and capital are linear in output. Equation (15) shows that the bubbly steady-state infrastructure level is financed by output taxes and the land-sale revenue generated by the housing bubble.

The following proposition characterizes the global equilibrium dynamics.

**Proposition 2** *Consider an economy with given initial condition  $\{K_0, A_0, H_0\}$ . If  $z \geq 1$  where  $z$  is given in (12), then no bubbly equilibrium exists. Otherwise, there is a unique  $\hat{Q}_0 > 0$  such that*

- (i) *if  $0 < Q_0 < \hat{Q}_0$ , then a bubbly equilibrium exists in which  $\lim_{t \rightarrow \infty} Q_t = 0$ ;*
- (ii) *if  $Q_0 = \hat{Q}_0$ , then a bubbly equilibrium exists in which  $Q^b \equiv \lim_{t \rightarrow \infty} Q_t > 0$ ;*
- (iii) *if  $Q_0 > \hat{Q}_0$ , then no bubbly equilibrium exists.*

To understand the intuition for Proposition 2, we explain how the long-run housing price,  $\lim_{t \rightarrow \infty} Q_t$ , depends on the initial  $Q_0$ . With a higher  $Q_0$ , more private capital  $K_1$  is crowded out, and capital return  $R_1$  becomes higher due to the diminishing marginal product of capital. The no-arbitrage condition (2) then implies a higher growth rate  $Q_1/Q_0$ . Using this argument for all the future dates, we conclude that higher  $Q_0$  raises the growth rate  $Q_{t+1}/Q_t$  for all  $t$ . If  $Q_0$  is sufficiently high, then the housing price explodes and cannot be sustained in equilibrium. If  $Q_0$  is sufficiently low, then the housing price declines to zero in the long run. There is a unique value  $\hat{Q}_0 > 0$  such that when  $Q_0 = \hat{Q}_0$ , the housing price converges to a positive limit.

### 3.7 Inspecting the Mechanism

In Tirole (1985) and Chen and Wen (2017), a bubble crowds out private capital and lowers output in the steady state. This is the traditional *crowding-out* effect of bubble. In our model, however, a housing bubble also helps the government accumulate more infrastructure, thus raising the productivity and production of the nonhousing sector. This is the *crowding-in* effect introduced in

our paper. Moreover, there is a factor *reallocation effect* in our model in that a housing bubble causes the labor used in the nonhousing sector to flow into the housing sector. We study these three effects in this subsection.

First we compare the steady-state infrastructure, capital, and output with and without bubble.

**Proposition 3** *We have the following relationships in the bubbly and bubbleless steady states:*

$$\begin{aligned}\frac{K^b}{K^n} &= (1 - \phi^b) \frac{Y^b}{Y^n}, \\ \frac{A^b}{A^n} &= \left(1 + \frac{\alpha_l \delta_h \phi^b \beta \alpha \psi (1 - \tau)}{(1 + \beta) \tau}\right) \frac{Y^b}{Y^n}, \\ \left(\frac{Y^b}{Y^n}\right)^{1 - \alpha - (1 - \rho)\theta} &= \left(1 + \frac{\alpha_l \delta_h \phi^b \beta \alpha \psi (1 - \tau)}{(1 + \beta) \tau}\right)^\theta (1 - \phi^b)^{\alpha - \rho\theta} (N_c^b)^{1 - \alpha - (1 - \rho)\theta},\end{aligned}\tag{17}$$

where  $N_c^b$  is given by (14).

From this proposition we can see that

$$\frac{K^b}{K^n} < \frac{Y^b}{Y^n} < \frac{A^b}{A^n}.$$

Thus, it is possible that  $Y^b > Y^n$ , but  $K^b < K^n$ . That is, the crowding-out effect on capital is dominated by the crowding-in effect on infrastructure. If the expression on the right-hand side of (17) is greater than 1, then  $Y^b > Y^n$ . This expression gives the determinants of  $Y^b/Y^n$ .

Under Assumption 1, a higher  $\theta$  strengthens the crowding-in effect because it increases the sensitivity of output to infrastructure. The first term on the right-hand side of (17) captures the impact of infrastructure funded by the land sale, while  $-\phi^b$  in  $(1 - \phi^b)^{\alpha - \rho\theta}$  captures the crowding-out effect on capital. The last term related to  $N_c^b < 1$  captures the reallocation effect on labor. If there is no housing bubble in the basic model, we have  $N_c^b = 1$  because the housing sector does not exist.

In general the above three effects are time varying along a transition path. The next two numerical examples illustrate these time-varying effects by comparing equilibrium paths with and without bubble. Both paths start from the same initial condition  $(A_0, K_0)$  and the land supply is fixed at  $L^*$  in each period.

**[Insert Figure 2 Here.]**

In the first example, we set  $(A_0, K_0) = (A^n, K^n)$ . Then the bubbleless equilibrium stays forever in the bubbleless steady state. Choose parameter values such that  $Y^b > Y^n$ . Figure 2 presents the transition dynamics. Panel A shows that bubbly output  $Y_t^b$  in the nonhousing sector is slightly lower at  $t = 0$  due to the reallocation effect on labor, but eventually higher than bubbleless output

$Y^n$ . Panel B shows the crowding-in effect of the housing bubble on infrastructure, and Panels C and D show the crowding-out effect on capital and the reallocation effect on labor. The crowding-out and reallocation effects dominate during the early stage of the transition. But the crowding-in effect gradually catches up and eventually dominates.

In the second example, we set small initial values for  $A_0$  and  $K_0$ , which are all smaller than the bubbleless steady-state values. Choose parameter values such that  $Y^b < Y^n$ . The bottom four panels of Figure 2 present the transition dynamics. We find that nonhousing output  $Y_t^b$  in the bubbly equilibrium is initially ( $t = 0$ ) lower than  $Y_t^n$  in the bubbleless equilibrium due to the reallocation effect on labor. From  $t = 1$  until  $t = 49$ ,  $Y_t^b$  is higher than  $Y_t^n$  because the crowding-in effect on infrastructure dominates the crowding-out effect on capital and the reallocation effect on labor. But  $Y_t^b$  is eventually lower than  $Y_t^n$  in the long run as the crowding-in effect is dominated.

To summarize, our basic model has illustrated the traditional crowding-out effect of housing bubble, the reallocation effect on labor, and a new crowding-in effect associated with the land sale by the Chinese local governments. To quantify these effects on the macroeconomy, we will calibrate our model in the next section.

## 4 Quantitative Analysis

To confront China's data, we extend our basic model in several ways. In particular, we introduce population growth and technology growth. We allow the housing asset to pay rents and thus its fundamental value is nonzero. The presence of a bubble permits housing prices to grow much faster than rents. We also introduce a stochastic housing bubble to conduct counterfactual experiments (Weil (1987)). We then calibrate this extended model and analyze its quantitative predictions. Appendix B presents the technical details for this model.

### 4.1 Stochastic Bubbles

Assume that all agents have common beliefs that the housing price is random and follows a two-state Markov process. In the bubbly state the housing price contains a bubble component. The bubble collapses with probability  $p_t$  in period  $t$ . Unlike Weil (1987) we allow the bursting probability  $p_t$  to vary over time. After the bubble collapses, the economy enters the fundamental state and stays there forever. The housing bubble cannot reappear. Given rational expectations, all equilibrium variables are stochastic and contingent on the state. There is no other shock in the model. When necessary, we use a variable with superscript  $+$  ( $-$ ) to denote its value in the bubbly (fundamental) state.

Assume that both workers and entrepreneurs live for  $T > 2$  years, and workers retire at age  $J$ . The population of both workers and entrepreneurs grows at a constant rate  $g_n$ . A newborn worker



of age 1 solves the following utility maximization problem:

$$\begin{aligned}
\max \quad & E \left[ \sum_{j=1}^T \beta^{j-1} \log(c_j^w) \right] \\
s.t. \quad & c_j^w + b_{j+1}^w = \begin{cases} w + R^f b_j^w, & 1 \leq j \leq J; \\ R^f b_j^w, & J+1 \leq j \leq T, \end{cases} \\
& b_1^w = 0, \quad b_{T+1}^w = 0,
\end{aligned}$$

where  $c_j^w$  is age- $j$  worker's consumption and  $b_j^w$  denotes bonds held at the beginning of age  $j$ . Here the expectation is taken with respect to the probability distribution of the stochastic bubble. A newborn worker does not have any asset so that  $b_1^w = 0$ . He/she does not leave any debt/asset when he/she dies. After retirement, the worker has no labor income and accumulates wealth from savings only. For simplicity, we have removed the time subscripts for all variables without risk of confusion.

A newborn entrepreneur in period  $t$  has initial endowment  $m_t$  and chooses his/her life-time consumption and investment in bonds, capital, and housing. The endowment  $m_t$  comes from a fraction  $\psi$  of the firms' after-tax profits. Housing delivers exogenous rents  $r_t$ , which grow at the rate  $g_r$ . Since the rental market is underdeveloped in China, we do not endogenize rents for simplicity. The entrepreneur's utility maximization problem is given by

$$\begin{aligned}
\max \quad & E \left[ \sum_{j=1}^T \beta^{j-1} \log(c_j^e) \right] \\
s.t. \quad & Qh_{j+1} + k_{j+1} + c_j^e + b_{j+1}^e \\
& = \begin{cases} m, & j = 1; \\ Rk_j + (Q(1 - \delta_h) + r)h_j + R^f b_j^e, & 2 \leq j \leq T; \end{cases} \\
& b_{j+1}^e \geq -\xi k_{j+1}, \\
& k_1 = b_1^e = h_1 = h_{T+1} = k_{T+1} = b_{T+1}^e = 0,
\end{aligned} \tag{18}$$

where  $c_j^e$  denotes an age- $j$  entrepreneur's consumption, and  $h_j$  and  $k_j$  are, respectively, his/her holdings of housing and capital at beginning of age  $j$ . Again we have removed the time subscripts. Notice that we allow entrepreneurs to save or borrow in the international financial market up to some borrowing limit. An entrepreneur can borrow against at most a fraction  $\xi$  of his/her capital assets. We assume that entrepreneurs cannot use residential housing as collateral because such a practice is uncommon in China (see Wu et al. (2015)).

We make three changes to the firms' production technologies in the extended model. First, we introduce labor-augmenting technology growth to match the Chinese economic growth. Let the labor efficiency  $e_t = (1 + g_e)^t$  grow exogenously at the rate  $g_e$ . Second, we introduce capital to the housing production function. Assume that both capital and labor are freely mobile across the housing and nonhousing sectors. Third, we allow the land quality to decline over time at the rate  $g_l$ .

The reason is that newly supplied land generally has less preferred location (Davis and Heathcote (2007) and Fang et al. (2015)). Real estate developers first build houses in cities and then build houses outside cities over time during the Chinese urbanization process. The quality of land in cities is better than that in rural areas as housing prices in cities grow much faster than in rural areas (Fang et al. (2015)).

Formally, let the production function in the housing sector be given by

$$y_{h,t} = ((1 - g_l)^t l_t)^{\alpha_l} (k_{h,t})^{\alpha_k} (e_t n_{h,t})^{1 - \alpha_l - \alpha_k},$$

where  $l_t$  denotes the land input and  $k_{h,t}$  denotes the capital input in the housing sector.

Each entrepreneur after age 1 runs both a nonhousing firm and a housing firm, and maximizes total profits:

$$\begin{aligned} R_t k_t = \max_{n_{c,t}, k_{c,t}, n_{h,t}, k_{h,t}, l_t} & \left\{ (1 - \psi) [(1 - \tau) (\hat{A}_t)^\theta (k_{c,t})^\alpha (e_t n_{c,t})^{1 - \alpha} - w_t n_{c,t}] \right. \\ & + (1 - \tau_h) Q_t ((1 - g_l)^t l_t)^{\alpha_l} (k_{h,t})^{\alpha_k} (e_t n_{h,t})^{1 - \alpha_l - \alpha_k} - w_t n_{h,t} - p_{L_t} l_t \\ & \left. + (1 - \delta_k) k_{c,t} + (1 - \delta_k) k_{h,t} \right\} \\ \text{s.t.} & \quad k_{c,t} + k_{h,t} = k_t, \end{aligned}$$

where  $k_t$  is total demand for capital across the two sectors,  $\delta_k$  is the depreciation rate of capital, and  $\tau_h$  is the tax rate in the housing sector. Here productivity satisfies

$$\hat{A}_t = \frac{A_t}{K_{c,t}^\rho (e_t N_{c,t})^{1 - \rho}},$$

where  $K_{c,t}$  and  $N_{c,t}$  denote the aggregate capital stock and aggregate labor in the nonhousing sector.

We allow the government to borrow at rate  $R^f$  in the extended model. Since more than half of the local government debt in China uses the land-sale revenue as collateral, we assume the amount of borrowing,  $B_{t+1}^g$ , is proportional to the land-sale revenue, i.e.,  $B_{t+1}^g = \xi_g p_{L_t} L_t$ , where  $\xi_g > 0$  is a parameter. The government uses debt, taxes and land-sale revenue to finance infrastructure investment  $A_{t+1} - (1 - \delta_a) A_t$  and non-infrastructure expenditure  $G_t$ . The government budget constraint is given by

$$\begin{aligned} & A_{t+1} - (1 - \delta_a) A_t + G_t + R^f B_t^g - B_{t+1}^g \\ & = \tau (\hat{A}_t)^\theta (K_{c,t})^\alpha (e_t N_{c,t})^{1 - \alpha} + \tau_h Q_t ((1 - g_l)^t L_t)^{\alpha_l} (K_{h,t})^{\alpha_k} (e_t N_{h,t})^{1 - \alpha_l - \alpha_k} + p_{L_t} L_t, \end{aligned} \quad (19)$$

where  $K_{h,t}$  and  $N_{h,t}$  are the aggregate capital and labor in the housing sector.

GDP in this economy is the sum of aggregate nonhousing output  $Y_t$ , residential investment (or the value of aggregate housing output)  $Q_t Y_{h,t}$ , and aggregate rents  $r_t H_t$ :

$$Y_t + Q_t Y_{h,t} + r_t H_t = (\hat{A}_t)^\theta (K_{c,t})^\alpha (e_t N_{c,t})^{1 - \alpha} + Q_t ((1 - g_l)^t L_t)^{\alpha_l} (K_{h,t})^{\alpha_k} (e_t N_{h,t})^{1 - \alpha_l - \alpha_k} + r_t H_t. \quad (20)$$

## 4.2 No-Arbitrage Pricing Equation

To understand the dynamics of housing prices, it is important to derive the pricing equation for the housing asset. When the economy is in the fundamental state in period  $t$ , it stays in this state forever. Under binding collateral constraints (which happens when  $R_{t+1}^- > R^f$ ), we can derive the following no-arbitrage condition:

$$\tilde{R}_{t+1}^- = \frac{Q_{t+1}^-(1 - \delta_h) + r_{t+1}}{Q_t^-}, \quad (21)$$

where the variable

$$\tilde{R}_{t+1}^- \equiv \frac{R_{t+1}^- - \xi R^f}{1 - \xi}$$

is the *effective capital return*, which takes into account the impact of the collateral constraint.<sup>7</sup> Equation (21) says that the housing return is equal to the effective capital return. Thus, in the fundamental state,  $Q_t^-$  is equal to the fundamental value, i.e., the present discounted value of future rents

$$Q_t^- = \sum_{s=t+1}^{\infty} \frac{(1 - \delta_h)^{s-(t+1)} r_s}{\prod_{j=t+1}^s \tilde{R}_j^-}.$$

Suppose that the economy is in the bubbly state in period  $t$ . Then the housing return is  $R_{t+1}^{h+} \equiv [(1 - \delta_h)Q_{t+1}^+ + r_{t+1}] / Q_t^+$  when the economy still stays in this state in period  $t+1$ . But the housing return is  $R_{t+1}^{h-} \equiv [(1 - \delta_h)Q_{t+1}^- + r_{t+1}] / Q_t^+$  when the economy moves to the fundamental state in period  $t + 1$ . Similarly, we can compute the effective capital returns under binding borrowing constraints:  $\tilde{R}_{t+1}^- \equiv (R_{t+1}^- - \xi R^f) / (1 - \xi)$  and  $\tilde{R}_{t+1}^+ \equiv (R_{t+1}^+ - \xi R^f) / (1 - \xi)$ . The no-arbitrage condition in period  $t$  is given by

$$\begin{aligned} & (1 - p_{t+1})u'(c_{j,t+1}^{e+})R_{t+1}^{h+} + p_{t+1}u'(c_{j,t+1}^{e-})R_{t+1}^{h-} \\ &= (1 - p_{t+1})u'(c_{j,t+1}^{e+})\tilde{R}_{t+1}^+ + p_{t+1}u'(c_{j,t+1}^{e-})\tilde{R}_{t+1}^-, \end{aligned} \quad (22)$$

where  $c_{j,t+1}^{e-}$  and  $c_{j,t+1}^{e+}$  are age- $j$  entrepreneur's consumption at  $t+1$  in the fundamental and bubbly states, respectively. Equation (22) says that the expected utility-adjusted housing return is equal to the expected utility-adjusted effective capital return as housing bubbles can collapse randomly.

## 4.3 Calibration

To calibrate parameter values, we simulate our extended model based on the equilibrium paths when the economy is always in the bubbly state. Suppose that the model economy starts in 2003 and one period in the model corresponds to one year. We focus on the sample period 2003-2013, during which the national housing-price data are available in China (Fang et al. (2015)). Some parameters are set exogenously, while the rest are estimated within the model.

<sup>7</sup>Substituting the binding collateral constraints into the budget constraint yields  $Q_t^- h_{j+1,t+1}^- + (1 - \xi)k_{j+1,t+1}^- + c_{j,t}^{e-} = (R_t^- - \xi R^f)k_{j,t}^- + (Q_t^- (1 - \delta_h) + r_t) h_{j,t}^-$ , which gives the expression for the effective capital return  $\tilde{R}_{t+1}^-$ .

Parameter	Description
$R^f = 1.003$	Annual interest rate
$g_n = 0.005$	Growth of population
$g_r = 0.005$	Growth of rents
$\tau = 0.14$	Tax rate in nonhousing sector
$\tau_h = 0.16$	Tax rate in housing sector
$\alpha_l = 0.56$	Land income share in housing sector
$\alpha_k = 0.22$	Capital income share in housing sector
$\alpha = 0.54$	Capital income share in nonhousing sector
$\theta = 0.1$	Output elasticity of infrastructure
$\rho = 0.5$	Capital congestion elasticity
$\zeta_b = 0.46$	Share of government expenditure in debt
$\kappa = 0.53$	Share of infrastructure investment in land-sale revenue
$\delta_h = 0.014$	Housing depreciation rate
$\delta_k = 0.1$	Capital depreciation rate
$\delta_a = 0.095$	Infrastructure depreciation rate

Table 2: Parameters estimated outside the model

We start by discussing parameters chosen exogenously as listed in Table 2. The interest rate  $R^f$  is set as 1.003, matching the average one-year real deposit rate (Song et al. (2011)). Similar to Song et al. (2011) and Chen and Wen (2017), agents enter the economy at age 22 and live for  $T = 50$  years, which is consistent with the average life expectancy of 71.4 years from the 2000 Chinese Population Census. Workers retire after working for 30 years. The population growth rate is set to  $g_n = 0.5\%$ , the average population growth rate during 2003-2013 from the National Bureau of Statistics of China (NBSC) data set. The growth rate of rents is set to  $g_r = 0.5\%$ , the average growth rate of rents during 2003-2013 according to the NBSC data set. Tax rates in the housing and nonhousing sectors are  $\tau_h = 0.16$  and  $\tau = 0.13$ , according to Bai et al. (2006).

We need to identify the housing sector in the data. In the model,  $Q_t Y_{ht}$  and  $Y_t$  represent the value added of the housing and nonhousing sectors, respectively. In the data, we interpret  $Q_t Y_{ht}$  as aggregate residential investment and  $Y_t$  as the remainder in the Chinese GDP.<sup>8</sup> Residential investment consists of land-sale revenue, capital income, and labor income in the real estate sector. The share of land-sale revenue  $\alpha_l = 0.56$  is set to match the average ratio of the land-sale revenue to residential investment. The capital income share in the housing sector  $\alpha_k = 0.24$  is computed using China's input-output table.<sup>9</sup>

Similarly, we calibrate the capital income share in the nonhousing sector as  $\alpha = 0.54$ . The

<sup>8</sup>The official data for the growth rates of rents are available, but the data for the level of rents are not. Since rents account for a very small share in GDP, we simply ignore them.

<sup>9</sup>First, we use capital income share in the construction sector to approximate  $\alpha_k$  in our model, as more than two thirds of production in China's construction sector is housing construction. Second, we compute capital share in the construction sector as follows. Although China's input-output table reports capital share  $\alpha_i$  for the value added in each sector  $i$ , we cannot use  $\alpha_{\text{constr}}$  directly because construction also uses intermediate goods such as steel and glass.

Parameter	Description	Target
$\beta = 0.999$	Discount factor	Average saving rate
$\psi = 0.42$	Wealth transfer share	Capital return in 2003
$\xi = 0.17$	Leverage ratio of firm	Average capital investment to GDP ratio
$g_e = 0.036$	Growth of labor efficiency	Average GDP growth rate
$g_l = 0.08$	Diminishing speed of land quality	Average residential investment to GDP ratio
$p_0 = 0.24$	Probability of bubble burst in 2003	Average housing price growth during 2003-2008
$\eta = 0.095$	Decay rate of burst probability	Average housing price growth during 2009-2013
$\zeta_y = 0.1$	Government expenditure/GDP ratio	Average infrastructure investment to GDP ratio
$\xi_g(t) = 2.37$ , if $t < 7$	Leverage ratio of government	Average local government debt to GDP ratio during 2003-2008
$\xi_g(t) = 3$ , if $t \geq 7$	Leverage ratio of government	Average local government debt to GDP ratio during 2009-2013
$K_0 = 1$	Initial capital stock	Output to capital ratio in 2003
$A_0 = 0.37$	Initial infrastructure stock	Infrastructure to capital ratio in 2003
$H_0 = 0.15$	Initial housing stock	Housing stock to capital ratio in 2003
$r_1 = 0.06$	Initial rent	Residential investment to GDP ratio in 2003

Table 3: Parameters calibrated in the model

elasticity of infrastructure is set to  $\theta = 0.1$ , which is estimated by Bom and Ligthart (2014). Since our model result is insensitive to  $\rho$ , we simply set  $\rho = 0.5$  (i.e., we assume capital and labor have the same congestion effect). Finally, we set depreciation rates as  $(\delta_k, \delta_a, \delta_h) = (0.10, 0.095, 0.0014)$  to match the average lifespan of equipment, structures, and residential housing of 12, 38, and 70 years in China, respectively, according to Bai et al. (2006).

Now we choose the remaining parameter values within the model to match certain data moments over 2003-2013 (see Table 3). We set  $\beta = 0.999$  to match the average saving rate of 48% in China,  $\psi = 0.42$  to match the post-tax capital return of 15% in 2003,  $\xi = 0.17$  to match the average

We define capital share  $\alpha_k$  as

$$\alpha_k \equiv \sum_{j=1}^N g_{\text{constr},j} \alpha_j,$$

where  $g_{\text{constr},j}$  is value added of intermediate goods from sector  $j$  divided by the total output of the construction sector. Third, we compute  $g_{\text{constr},j}$  as follows. In the input-output table,  $X = AX + d$ , where  $X = [X_i]$  is the vector of outputs for each sector,  $d = [d_i]$  is the vector of value added, and  $A$  is the direct consumption coefficient matrix. Therefore,  $X = Bd$ , where  $B = (I - A)^{-1}$ , and  $X_{\text{constr}}$  is decomposed as  $X_{\text{constr}} = \sum_{j=1}^N b_{\text{constr},j} d_j$ . Therefore,  $g_{\text{constr},j} = \frac{b_{\text{constr},j} d_j}{X_{\text{constr}}}$ .

investment rate of 42%, and the growth rate of labor efficiency  $g_e = 0.036$  to match the average GDP growth rate of 10% during 2003-2013. While the long-run GDP growth rate is equal to  $(1 + g_n)(1 + g_e) - 1 = 4.1\%$ , the average growth rate during the transition period can be much higher.

Since we will conduct counterfactual experiments in Section 5, we need to calibrate land supply  $L_t$  beyond the period 2003-2013. We choose  $L_t$  for  $0 \leq t \leq 13$  to match the land supply in 2003-2016 taken from the China Land Statistical Yearbook. We normalize the land supply in 2003 to 1 so that  $L_0 = 1$ . We do not have the land supply data starting from 2017. We assume that the quantity of land supply since 2017 is a constant equal to the average land supply during 2003-2016. As pointed out by Davis and Heathcote (2007), to get constant-quality land supply, the quantity of land supply need to be adjusted by the quality. We assume that the labor quality declines at the rate  $g_l$ . We calibrate  $g_l = 0.08$  to match the average ratio of the residential investment to GDP in 2003-2013.

To study the impact of a bubble burst, we introduce a stochastic bubble similar to Weil (1987). Unlike Weil (1987), we assume that the probability of the bubble burst is a time-varying function  $p_t = p_0(1 - \eta)^t$ . Under our specification, the bursting probability declines to zero in the long run. This can generate high housing price growth during the transition period for people to hold the risky bubbly housing asset. Moreover, it also allows us to match the high capital return in the data, as the capital return in the bubbly steady state is higher than in the bubbleless steady state. We set  $(p_0, \eta) = (0.24, 0.095)$  to match the average growth rates of the housing price during 2003-2008 and 2009-2013.

In Appendix B we show that, on the bubbly balanced growth path, the rent to housing price ratio converges to zero, and the housing price and its bubble component grow at the same rate:

$$\left( \frac{(1 + g_e)(1 + g_n)}{1 - g_l} \right)^{\alpha_l} - 1 = 7.2\%,$$

which is higher than the long-run GDP growth rate of 4.1%, according to our calibration. Our model also implies that the housing price can grow much faster during the transition period due to the growing bubble. Time-varying bursting probabilities also cause high housing price growth as entrepreneurs require a high return to hold the risky housing asset.

The Chinese local government debt to GDP ratio increased over 2003-2013.<sup>10</sup> In particular, the average local government debt to GDP ratio increased from 9% before 2009 to 17% after 2009. This is because after the U.S. financial crisis in 2009, the Chinese central government implemented a large economic stimulus package, the so-called Four Trillion Project. Over three-fourth of the expenditure was financed by the local government debt (Bai et al. (2016)). China prohibited local

---

<sup>10</sup>Our local government debt data are from the Audit Report on National Government Debt 2011 and 2013 issued by the National Audit Office of China (NAOC). The data only include the debt that the local governments are guaranteed to pay back, but excludes the local governments contingent liability.

governments from issuing bonds before 2009, but relaxed this regulation in 2009. This policy change caused fast growing local government debt, more than half of which was backed by the land-sale revenue. In our calibration, we set  $\xi_g = 2.37$  before 2009 and  $\xi_g = 3$  after 2009 to match the average ratio of the local government debt to GDP before and after 2009.

The government non-infrastructure expenditure  $G_t$  is financed through three sources: output, government debt, and land-sale revenue. We specify the following rule:

$$G_t = \zeta_y Y_t + \zeta_b (B_{t+1}^g - R^f B_t^g) + (1 - \kappa) p_{Lt} L_t.$$

Since 54% of the local government debt was spent on infrastructure investment and the remainder was spent on other expenditure as reported by the National Audit Office of China, we set  $\zeta_b = 0.46$ . Assume that only a fraction  $\kappa$  of the land-sale revenue is used to finance infrastructure investment and the remaining fraction  $1 - \kappa$  is used to finance non-infrastructure expenditure. We set  $\kappa = 0.53$  to match the average share of infrastructure investment in the land-sale revenue in the data. To calibrate  $\zeta_y$ , we notice that an increase in  $\zeta_y$  raises the non-infrastructure expenditure and thus reduces the infrastructure investment expenditure by the government budget constraint (19). Then we choose  $\zeta_y = 0.1$  to match the average infrastructure investment to GDP ratio (7.5%).

We set the initial condition in the model as follows. First, we normalize the initial population and labor efficiency to 1. Second, we calibrate the initial  $(K_0, A_0, H_0)$  to  $(1, 0.37, 0.15)$  to match the capital-output ratio, the housing-capital ratio, and the infrastructure-capital ratio in 2003.<sup>11</sup> Third, the initial housing rent is set to  $r_1 = 0.06$  to match the ratio of the residential investment to GDP in 2003. Finally, following Song et al. (2011), the initial wealth distribution of entrepreneurs across various generations is set as the wealth distribution of workers in the steady state.

#### 4.4 Results

Figure 3 presents the data and results based on our calibrated model. While our model is targeted to match either the average values over 2003-2013 or the initial values in 2003 in the data, our model can match both the first moments and the dynamic patterns in the data fairly well due to our model mechanism. In particular, the rapid rise of housing prices is associated with the increases of the infrastructure investment to GDP ratio, the land-sale revenue to GDP ratio, and the residential investment to GDP ratio (see Panels D, E, and F). We leave the discussion of GDP growth to the next subsection. Our model does not match the cyclicalities of the data shown in Figure 3. These cyclicalities may be due to various business cycle shocks and uncertainties about China's monetary, fiscal, and housing market policies. Our model does not incorporate these features (except for the risk of the bubble burst) and hence it cannot match the cyclical pattern in the data.

<sup>11</sup>Bai et al. (2006) show capital-output ratio is 1.66 in 2003 and 13% of capital is residential housing. Jin (2016) shows 25% of total capital is infrastructure in 2003. Note that capital in these studies refers to commercial capital, infrastructure, and housing, while capital  $K_t$  in our paper is only commercial capital.

[Insert Figure 3 Here.]

The increase of the residential investment to GDP ratio over time in the model ( $Q_t Y_{ht}/GDP_t$ ) is due to two effects: the rapid rise of housing prices  $Q_t$  and the reallocation of capital and labor to the housing sector. The reallocation effect causes  $K_{h,t}$  and  $N_{h,t}$  to rise so that  $Y_{h,t}$  increases. Since the land-sale revenue  $p_{Lt}L_t$  is equal to  $\alpha_l(1 - \tau_h)(Q_t Y_{h,t})$  due to the Cobb-Douglas production function, the land-sale revenue rises with the residential investment proportionally in the model.

As the land-sale revenue increases over time, the government can finance more infrastructure investment so that the infrastructure investment to GDP ratio rises over time as in the data. Notice that infrastructure investment rises dramatically in 2009 both in the model and in the data. This is due to the Four Trillion Project in 2009 when the Chinese local government used government debt backed by the land-sale revenue to finance infrastructure investment. Our model captures this event by raising the government leverage ratio  $\xi_g$  from 2.37 to 3 starting in 2009. We are able to match the average ratio of the local government debt to GDP during 2003-2013, see Figure 4.

[Insert Figure 4 Here.]

While infrastructure raises productivity, the capital return is high and declines over time (see Figure 3 Panel B). Recall that the capital return in our model satisfies

$$R_t = \alpha(1 - \tau)(1 - \psi)\hat{A}_t^\theta K_{c,t}^{\alpha-1}(e_t N_{c,t})^{1-\alpha} + (1 - \delta_k).$$

The decline is due to the diminishing marginal product of capital during the transition period. This effect dominates the increase in infrastructure as capital is accumulated over time.

Now we discuss housing prices presented in Figure 3 Panel C. Our model matches the growing trend of China's housing prices fairly well. Our simulated growth rate of housing prices drops from 12.9% in 2003 to 9% in 2013. This decrease is only half of the decrease in the capital return. While the capital return follows a downward trend, the growth rates of housing prices are quite stable.

It seems puzzling that the growth rate of housing prices stays high on average, while the capital return follows a fast downward trend. To understand the intuition, consider the no-arbitrage equation (22) under risk-neutral utility and zero depreciation of housing ( $u'(c) \equiv 1$  and  $\delta_h = 0$ ):

$$\bar{R}_{t+1} = \frac{r_{t+1}}{Q_t^+} + \frac{p_{t+1}Q_{t+1}^- + (1 - p_{t+1})Q_{t+1}^+}{Q_t^+}, \quad (23)$$

where  $\bar{R}_{t+1}$  denotes the (expected effective) capital return. That is, the expected capital return is equal to the expected housing return, which in turn is equal to the sum of the rent-to-price ratio and expected price appreciation. The rent-to-price ratio is relatively high initially in 2003 and the size of the bubble is also small (i.e.,  $Q_{t+1}^-$  is close to  $Q_{t+1}^+$ ). Thus the housing price growth rate  $Q_{t+1}^+/Q_t^+$  is approximately equal to  $\bar{R}_{t+1} - r_{t+1}/Q_t^+$ , which is less than the capital return  $\bar{R}_{t+1}$  in



the early period of 2003-2013. As time goes by, both the housing price and the housing bubble grow, but the rents grow at a much lower rate (about 0.5% on average in the data). The rent-to-price ratio gradually declines to zero in the long run. Thus the fundamental value of housing  $Q_t^-$  relative to the bubbly value  $Q_t^+$  approaches zero. In this case, equation (23) implies that the housing price growth rate  $Q_{t+1}^+/Q_t^+$  approaches  $\bar{R}_{t+1}/(1-p_t)$ , which is greater than the capital return  $\bar{R}_{t+1}$  for  $p_t > 0$ . This happens in the later years of 2003-2013 in our model. Intuitively, to compensate the risk of the bubble burst, the growth rate of housing prices when the bubble never bursts must be higher than the capital return (Weil (1987)).

To close this section, we argue that the land-sale revenue channel is essential for our analysis. To see this, let us simply shut down the land-sale revenue channel by assuming that the land-sale revenue is transferred to workers without recalibrating the model. We find that the average infrastructure investment to GDP ratio is lowered by 5.7 percentage points, and the average GDP growth rate is lowered by 1 percentage point, compared to our extended model. Moreover we cannot match the increasing pattern of the infrastructure investment to GDP ratio in the data. If we further recalibrate the government spending  $G_t$  to match the average infrastructure investment to GDP ratio, we still cannot match the upward trend of this ratio.

#### 4.5 Growth Accounting

In this subsection we discuss GDP growth. Panel A of Figure 3 shows that our model can replicate the average GDP growth rate of 10% during 2003-2013, as well as the drop from the highest growth rate of 14% to 7% during this period. To understand this pattern, we conduct a growth accounting exercise.

Recall that GDP is defined in equation (20). Then we can decompose GDP growth as

$$\frac{\Delta GDP_t}{GDP_t} \approx \frac{Y_t}{GDP_t} \frac{\Delta Y_t}{Y_t} + \frac{Q_t Y_{h,t}}{GDP_t} \frac{\Delta(Q_t Y_{h,t})}{Q_t Y_{h,t}} + \frac{r_t H_t}{GDP_t} \frac{\Delta(r_t H_t)}{r_t H_t}. \quad (24)$$

Our calibrated model shows that the average growth rate of residential investment is 16.2% and the average residential investment to GDP ratio is 8.6%, while the average nonhousing output growth rate is only 9.3% and the average nonhousing output to GDP ratio is 90%. Thus the 10% average GDP growth consists of 8.4% of nonhousing output growth and 1.4% of the residential investment growth. Rents only contribute to 0.2% of GDP growth on average and thus will be ignored in our discussions below.

Since aggregate output in the nonhousing sector satisfies

$$Y_t = A_t^\theta K_{c,t}^{\alpha-\rho\theta} (e_t N_{c,t})^{1-\alpha-(1-\rho)\theta},$$

we can further decompose its growth into

$$\begin{aligned}\frac{\Delta Y_t}{Y_t} &\approx \theta \frac{\Delta A_t}{A_t} + (\alpha - \rho\theta) \frac{\Delta K_{c,t}}{K_{c,t}} + (1 - \alpha - (1 - \rho)\theta) \frac{\Delta e_t}{e_t} \\ &\quad + (1 - \alpha - (1 - \rho)\theta) \frac{\Delta N_{c,t}}{N_{c,t}}.\end{aligned}$$

Based on our calibrated model, we find that

$$\begin{aligned}9.3\% &\approx 0.1 * 10.7\% + 0.49 * 13.8\% + 0.41 * 3.6\% + 0.41 * 0.3\% \\ &\approx 1.1\% + 6.7\% + 1.5\% + 0.1\%.\end{aligned}$$

Thus, to the 9.3% average nonhousing sector growth, infrastructure contributes 1.1%, capital 6.7%, labor efficiency 1.5%, and labor 0.1%.

Similarly, we can decompose the residential investment growth into

$$\begin{aligned}\frac{\Delta(Q_t Y_{ht})}{Q_t Y_{ht}} &\approx \frac{\Delta Q_t}{Q_t} + \alpha_l \left( \frac{\Delta L_t}{L_t} - g_l \right) + \alpha_k \frac{\Delta K_{h,t}}{K_{h,t}} + (1 - \alpha_l - \alpha_k) \frac{\Delta e_t}{e_t} \\ &\quad + (1 - \alpha_l - \alpha_k) \frac{\Delta N_{h,t}}{N_{h,t}}.\end{aligned}$$

Our calibrated model shows that

$$\begin{aligned}16.2\% &\approx 10.0\% + 0.56 * (-1.7\%) + 0.24 * 21\% + 0.2 * 3.6\% + 0.2 * 6.6\% \\ &\approx 10.0\% + (-1\%) + 5\% + 0.7\% + 1.3\%.\end{aligned}$$

Thus, to the 16.2% average housing sector growth, housing price contributes 10.0%, land  $-1\%$ , capital 5%, and labor 1.3%.

To see why GDP growth declined over 2003-2013 in the data, we separate the whole sample into two periods 2003-2008 and 2009-2013. The average GDP growth is 11.4% in the first period, and 8.6% in the second period. Using (24), we can decompose these growth rates into

$$\begin{aligned}11.4\% &\approx 92\% * 10.9\% + 7.2\% * 16.6\% + 1.2\% * 21.2\% \\ &\approx 10\% + 1.2\% + 0.3\%,\end{aligned}$$

and

$$\begin{aligned}8.6\% &\approx 88\% * 7.7\% + 10.2\% * 15.9\% + 1.6\% * 0.2\% \\ &\approx 6.8\% + 1.6\% + 0.2\%.\end{aligned}$$

This decomposition shows that the decline of GDP growth is attributed mainly to the decline of the nonhousing sector growth from 10% to 6.8%.

Table 4 presents the decomposition in terms of factor inputs. We find that the decline of the nonhousing sector growth is attributed mainly to the decline of the capital growth from 17.5% to 10.1%, while the infrastructure growth rises from 8.8% to 12.6%.

Variable (%)	$\Delta A/A$	$\Delta K_c/K_c$	$\Delta N_c/N_c$	$\Delta Q/Q$	$\Delta L/L - g_l$	$\Delta K_h/K_h$	$\Delta N_h/N_h$
2003-2008	8.8	17.5	0.3	11.0	-3.4	23.5	5.4
2009-2013	12.6	10.1	0.2	9.0	-0.3	18.4	7.8
2003-2013	10.7	13.8	0.3	10.0	-1.7	21.0	6.6

Table 4: Growth accounting based on the calibrated model.

## 5 Counterfactual Experiments

Due to the dramatic growing trend of housing prices, the Chinese policy makers and academic researchers are concerned that housing prices contain a bubble and the collapse of the bubble may damage the economy. There is also a discussion of implementing a property tax to control housing prices. In this section we use our calibrated model to study the impact of the collapse of a housing bubble and the impact of the property tax.

### 5.1 Bubble Burst

Suppose that the economy stays in the bubbly state until the housing bubble bursts in 2025. After the burst, the economy stays in the fundamental state forever. Figure 5 Panel A plots the growth rates of the housing price in the two economies with and without the bubble burst. Immediately after the burst, the growth rate of the housing price drops from 6.6% to  $-45.1\%$ . In the next 30 years, it is 1.2% on average, much lower than the average growth rate of 5.3% without the burst. This can be explained by the low rent growth rate of 0.5%. On a balanced growth path in the fundamental state, the growth rate of the bubbleless housing price is equal to the rent growth rate. Thus the average growth rate of the housing price is low during the transition period.

[Insert Figure 5 Here.]

Figure 5 Panel B shows how the bubble burst affects GDP. After the bubble bursts in 2025, the growth rate of GDP drops from 5% to 2.6%. This 2.6% GDP growth consists of a 6.8% increase of output in the nonhousing sector and a 68.8% decrease in the housing sector. Nonhousing output increases because capital and labor flow back from the housing sector into the nonhousing sector, while housing output decreases because newly built houses lose the bubble value. Despite the large drop of housing prices, its impact on GDP growth is relatively small because the housing sector accounts for a small share of GDP (5.4% without burst and 1.6% with burst). The rise of nonhousing output offsets the large decline of housing output.

One year after the bubble bursts, however, the GDP growth rate is higher than that when the bubble never bursts. In the next 30 years the average GDP growth rate after the burst is 0.8 percentage points higher. In the long run on the balanced growth path the GDP growth rates in

the two economies with and without bubble are both equal to the sum of the population growth and the technology growth. But the burst of the housing bubble has a level effect. In particular, in 2055, GDP and nonhousing output after the burst are 22.8% and 24.8% higher than those without the burst.

The reason that GDP in 2055 after the bubble bursts is higher than that without the burst is due to the following effects. After the burst of the bubble, infrastructure investment is reduced due to the decline of the land-sale revenue, but aggregate capital is unleashed: during 2025-2055, the average growth rate of infrastructure decreases from 3.2% to 3.1%, while the average growth rate of capital increases from 4.2% to 6.1%. Moreover, capital and labor are reallocated from the housing sector to the nonhousing sector (see Panels C and D of Figure 5). Since the nonhousing sector accounts for a much larger share of the economy, the increased nonhousing output raises GDP.

## 5.2 Property Tax

The Chinese government has not adopted a comprehensive property tax so far. In this subsection we suppose that the Chinese government initiates a permanent linear property tax on the entire housing stock in 2025. In our benchmark, the tax rate is 1.5% and this tax policy is unexpected by all agents in the model. Since there is a discussion among Chinese policy makers that the property tax revenue can be used to finance the local government spending on infrastructure investment, we simply assume that all property tax revenue is used to finance infrastructure investment. We focus on the equilibrium paths when housing bubbles never crash both before and after the tax policy.<sup>12</sup>

Figure 6 Panel A plots housing prices after the property tax is imposed. The property tax generates a negative wealth effect, which reduces entrepreneurs' housing demand. On impact, the housing price drops by 30.8% and its growth rate in 2025 drops from 6.6% to -26.2%. From 2026 to 2055, the average growth rate of housing prices is higher by 0.5 percentage points than that without the property tax. In the long run, the growth rate of housing prices with the property tax is the same as that without it.

**[Insert Figure 6 Here.]**

In Figure 6 Panel B, we show how the property tax changes GDP. Based on our simulation, after the property tax is imposed, GDP drops immediately by 1.6% compared with the case without the property tax. This 1.6% GDP drop consists of a 6.3% increase of output in the nonhousing sector and a 47.2% decrease in the housing sector. During 2026-2055 the average growth rate of GDP increases from 4% to 4.8%. In 2055, thirty years after the property tax is started, GDP and output in the nonhousing sector are 18.5% and 19.4%, respectively, higher than those without the property

---

<sup>12</sup>Miao et al. (2015) show that, when the property tax rate is sufficiently high, a housing bubble can never emerge.

tax. The reason that the long-run GDP with the property tax is higher than that without is due to the following three effects. First, the property tax encourages capital accumulation because entrepreneurs invest less in housing assets. The average capital growth increases from 4.2% to 5%. Second, the property tax also increases infrastructure accumulation as we have assumed that the tax revenue is used to finance infrastructure investment. The average infrastructure growth increases from 3.2% to 5.3%. Third, with the decline of housing prices, more capital and labor are reallocated from the housing sector to the nonhousing sector. Both the average capital share and labor share of the nonhousing sector increase by 0.7 percentage points.

### 5.3 Welfare Effects

In this subsection we study the welfare effects of the above two counterfactual experiments on both workers and entrepreneurs alive in 2025. First, consider the impact of the bubble burst in 2025, presented in Panel A of Figure 7. We measure the welfare change as a percentage deviation in the life-time consumption from the equilibrium in which the housing bubble never bursts.

[Insert Figure 7 Here.]

The oldest cohort living in 2025 enters the economy in 1976 at age 22 in the model. The bubble burst does not affect wages before 2025, but decreases the wage rate in 2025 because workers flow into the nonhousing sector. The wage rate rises after 2025 because more resources are allocated to capital accumulation so that the marginal product of labor rises. As a result, all workers born before 1996 in our model do not experience welfare changes as these cohorts of workers retire before the bubble bursts. Their lifetime income, which is the present value of wages, is unchanged. Workers born in 1996 suffer a tiny welfare loss as their wages decline in 2025 only. Workers born after 1996 experience welfare gains, because they enjoy an increase in the wage rate during their working periods. The younger the workers are, the larger their welfare gains due to their ability to work for a longer period of time.

By contrast, all entrepreneurs alive in 2025 suffer welfare losses. This is because of the permanent loss of the housing value and the decline of the capital return after the bubble bursts. The welfare jump of the latest cohort in 2025 is because the newborn entrepreneurs in our model do not hold any housing assets and start trading housing assets in subsequent years. Except for this cohort, the younger the entrepreneurs are, the longer they face declined capital returns, and therefore the larger their welfare losses.

There are two main differences between the welfare result of Chen and Wen (2017) and ours. First, the wage rate declines in the year when the bubble bursts in our model due to the labor reallocation effect, which is absent in Chen and Wen (2017). Second, the capital return in Chen and Wen (2017) is constant during the transition stage and the bubble burst lowers the capital

%	Benchmark	$\theta = 0.2$	$\rho = 0.75$	$\rho = 0.25$	$p_\infty = 0.01$
GDP growth	10.0	9.0	9.6	10.4	10.0
Housing price growth	10.0	8.9	9.7	10.3	9.6
Nonhousing sector growth	9.3	8.4	8.9	9.8	9.4
Housing sector growth	16.2	14.0	16.0	16.6	15.6
Capital growth	14.0	11.7	13.6	14.4	14.2
Infrastructure growth	10.7	9.7	10.6	10.8	10.5

Table 5: Sensitivity analysis.

return only in the post-transition stage. By contrast, the capital return in our model immediately drops so that the welfare losses of entrepreneurs predicted in our model are larger than in Chen and Wen (2017).

Next we consider the welfare effects of the property tax, presented in Panel B of Figure 7. We measure the welfare change as a percentage deviation in the life-time consumption from the equilibrium without a property tax to the equilibrium with the property tax studied earlier. We find that the result is similar to that discussed for the case of the bubble burst, except that the magnitude here is smaller. The intuition is also similar.

## 6 Discussions

In this section we conduct a sensitivity analysis. Since infrastructure plays an important role in our model, one may wonder whether our results are sensitive to changes in parameters such as the productivity elasticity parameter  $\theta$  and the congestion effect parameter  $\rho$ .

In our benchmark quantitative model we set the elasticity of infrastructure  $\theta = 0.1$  following Bom and Ligthart (2014). Now we double the value of  $\theta$ , holding all other parameter values fixed, and report the results in Table 5 column 2. We find that the average growth rates over 2003-2013 of GDP, housing prices, nonhousing output, housing output, aggregate capital, and infrastructure investment all decline with  $\theta$ . The reason is that a higher value of  $\theta$  increases not only infrastructure productivity but also the congestion effect of capital and labor. Since capital accumulation is the main driving force of Chinese GDP growth, the congestion effect dominates the increase in infrastructure productivity, causing GDP growth to slow down. This in turn causes all other growth rates reported in Table 5 column 2 to decline.

Next we report the sensitivity analysis of  $\rho$  in Table 5 columns 3 and 4, holding all other parameter values fixed. Although we choose  $\rho = 0.5$  in our benchmark model, our results are not sensitive to this choice. In particular, GDP growth decreases slightly in Table 5 when  $\rho$  increases. Higher  $\rho$  strengthens the congestion effect of capital, makes capital less productive, and drives down the GDP growth. But higher  $\rho$  also weakens the congestion effect of labor, which partially offsets

the former negative effect.

Finally, to save computation time, we have assumed that the housing bubble will never burst in the long run in our benchmark model. Here we assume that the bubble will eventually burst with probability 1%, i.e.,  $\lim_{t \rightarrow \infty} p_t = p_\infty = 0.01$ . Table 5 column 5 shows that the average housing price growth and the housing sector growth decline slightly, compared to our benchmark calibration. The average GDP growth, however, is unaffected because faster capital accumulation raises the nonhousing sector growth.

Notice that in all cases studied above, we have not recalibrated the model to match the same data moments as in our benchmark model. When we recalibrate the model, we find that the new results are almost identical to those in the benchmark.

## 7 Conclusion

In this paper we study the impact of Chinese housing bubbles on infrastructure investment and economic growth in a two-sector OLG model. Our calibrated model can match the Chinese data reasonably well. Our study makes three contributions. First, we identify a new crowding-in effect of housing bubbles, by introducing a land-sale channel unique to the Chinese economy. With this channel, our model can explain the boom of infrastructure investment in China. Second, we quantify the effects of a bubble burst and find that, although the crash represents a big negative shock to investors' wealth, the effect on China's real GDP is relatively small due to the reallocation effect of capital and labor. Third, imposing a property tax and use the tax revenue to finance infrastructure investment can lower housing prices and reallocate resources to the nonhousing sector, thereby raising the long-run GDP level.

We have not considered the collateral channel of housing prices in the nonhousing sector because this channel seems weak in the Chinese data (Wu et al. (2015)). If nonhousing firms use houses or land as collateral to borrow to finance capital investment, then changes in housing prices can have a large impact on nonhousing output and hence GDP (Kiyotaki and Moore (1997)). Given that small firms are more likely to use houses as collateral and their investment accounts for a small share of aggregate investment, we expect that incorporating the collateral channel would not change our results significantly. Further study of this issue would be an interesting topic for future research.

## References

- Arce, O. and D. López-Salido (2011). Housing bubbles. *American Economic Journal: Macroeconomics* 3, 212–241.
- Bai, C.-E., C.-T. Hsieh, and Y. Qian (2006). The return to capital in china. *Brookings Papers on Economic Activity* 37(2), 61–102.
- Bai, C.-E., C.-T. Hsieh, and Z. Song (2016). The long shadow of chinas fiscal expansion. *Brookings Papers on Economic Activity* 47(2), 129–181.
- Bai, C.-E. and J. Zhang (2014). Return to Capital in China and its Determinants. *World Economy* 10.
- Barro, R. (1990). Government spending in a simple model of endogenous growth. *Journal of Political Economy* 98(5), S103–26.
- Bassetto, M. and T. J. Sargent (2006). Politics and efficiency of separating capital and ordinary government budgets. *The Quarterly Journal of Economics* 121(4), 1167–1210.
- Baxter, M. and R. King (1993). Fiscal policy in general equilibrium. *American Economic Review* 83(3), 315–34.
- Bom, P. and J. Ligthart (2014). What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys* 28(5), 889–916.
- Chen, K. and Y. Wen (2017, April). The great housing boom of china. *American Economic Journal: Macroeconomics* 9(2), 73–114.
- Chen, P., C. Wang, and Y. Liu (2015). Real estate prices and firm borrowings: Micro evidence from china. *China Economic Review* 36, 296 – 308.
- Chen, T., L. X. Liu, W. Xiong, and L.-A. Zhou (2017). Real estate boom and misallocation of capital in china. *Working Paper*.
- Davis, M. and J. Heathcote (2007). The price and quantity of residential land in the united states. *Journal of Monetary Economics* 54(8), 2595–2620.
- Dong, F., J. Liu, Z. Xu, and B. Zhao (2019). Flight to housing in china. *working paper, Tsinghua University*.
- Fang, H., Q. Gu, W. Xiong, and L.-A. Zhou (2015). Demystifying the chinese housing boom. In *NBER Macroeconomics Annual 2015, Volume 30*, pp. 105–166. National Bureau of Economic Research, Inc.



- Farhi, E. and J. Tirole (2012). Bubbly liquidity. *Review of Economic Studies* 79(2), 678–706.
- Glomm, G. and B. Ravikumar (1994). Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control* 18, 1173–1187.
- Hirano, T. and N. Yanagawa (2016). Asset bubbles, endogenous growth, and financial frictions. *Review of Economic Studies* 84(1), 406–443.
- Iacoviello, M. (2005). House prices, borrowing constraints and monetary policy in the business cycle. *American Economic Review* 95(3), 739–763.
- Iacoviello, M. and S. Neri (2010). Housing market spillovers: Evidence from an estimated dsge model. *American Economic Journal: Macroeconomics* 2.
- Jin, G. (2016). Infrastructure and non-infrastructure capital stocks in china and their productivity: A new estimate. *Economic Research Journal* (5), 41–56.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–48.
- Liu, Z., P. Wang, and T. Zha (2013). Land-price dynamics and macroeconomic fluctuations. *Econometrica* 81(3), 1167–1184.
- Martin, A. and J. Ventura (2012). Economic growth with bubbles. *American Economic Review* 102(6), 3033–58.
- Miao, J. and P. Wang (2014). Sectoral bubbles, misallocation, and endogenous growth. *Journal of Mathematical Economics* 53, 153–163.
- Miao, J. and P. Wang (2018). Asset bubbles and credit constraints. *American Economic Review* 108(9), 2590–2628.
- Miao, J., P. Wang, and J. Zhou (2015). Asset bubbles, collateral, and policy analysis. *Journal of Monetary Economics* 76.
- Mo, J. (2018). Land financing and economic growth: Evidence from chinese counties. *China Economic Review* 50, 218–239.
- Song, Z., K. Storesletten, and F. Zilibotti (2011). Growing like china. *American Economic Review* 101(1), 196–233.
- Tirole, J. (1985). Asset bubbles and overlapping generations. *Econometrica* 53(6), 1499–1528.
- Weil, P. (1987). Confidence and the real value of money in an overlapping generations economy. *Quarter Journal of Economics* 102(1), 1–22.

- Wu, G. L., Q. Feng, and Z. Wang (2019). Estimating productivity of public infrastructure investment. *Working Paper, Nanyang Technological University*.
- Wu, J., Y. Deng, and H. Liu (2014). House price index construction in the nascent housing market: The case of china. *Journal of Real Estate Finance and Economics* 48(3), 522–545).
- Wu, J., J. Gyourko, and Y. Deng (2015). Real estate collateral value and investment: The case of china. *Journal of Urban Economics* 86, 43–53.
- Xiong, W. (2019). The mandarin model of growth. *Working Paper, Princeton University*.
- Zhao, B. (2015). Rational housing bubbles. *Economic Theory* 60(1), 141–201.

## Appendix

### A The Basic Model in Section 3

#### A.1 Equilibrium Dynamics

The bubbly equilibrium of the basic model can be summarized by the following system of 12 nonlinear difference equations for  $t \geq 0$ :

$$A_{t+1} = (1 - \delta_a)A_t + \tau Y_t + p_{Lt}L_t, \quad (\text{A.1})$$

$$H_{t+1} = (1 - \delta_h)H_t + Y_{ht}, \quad (\text{A.2})$$

$$K_{t+1} = \frac{\beta}{1 + \beta}M_t - Q_t H_{t+1}, \quad (\text{A.3})$$

$$R_{t+1} = \frac{Q_{t+1}(1 - \delta_h)}{Q_t}, \quad (\text{A.4})$$

$$R_t = \alpha(1 - \tau)(1 - \psi) \left[ A_t / (K_t^\rho N_{c,t}^{1-\rho}) \right]^\theta K_t^{\alpha-1} N_{c,t}^{1-\alpha}, \quad (\text{A.5})$$

$$w_t = (1 - \alpha)(1 - \tau) \left[ A_t / (K_t^\rho N_{c,t}^{1-\rho}) \right]^\theta K_t^\alpha N_{c,t}^{-\alpha}, \quad (\text{A.6})$$

$$w_t = (1 - \alpha_l)Q_t L_t^{\alpha_l} N_{h,t}^{-\alpha_l}, \quad (\text{A.7})$$

$$p_{Lt} = \alpha_l Q_t L_t^{\alpha_l-1} N_{h,t}^{1-\alpha_l}, \quad (\text{A.8})$$

$$1 = N_{c,t} + N_{h,t}, \quad (\text{A.9})$$

$$Y_t = \left[ A_t / (K_t^\rho N_{c,t}^{1-\rho}) \right]^\theta K_t^\alpha N_{c,t}^{1-\alpha}, \quad (\text{A.10})$$

$$Y_{h,t} = L_t^{\alpha_l} N_{h,t}^{1-\alpha_l}, \quad (\text{A.11})$$

$$M_t = \psi \alpha (1 - \tau) Y_t, \quad (\text{A.12})$$

for 12 sequences of aggregate variables

$$\{R_t, w_t, p_{Lt}, N_{c,t}, N_{h,t}, Y_t, Y_{h,t}, M_t, K_t, A_t, H_t, Q_t\}_{t=0}^\infty.$$

The variables  $A_t$ ,  $K_t$ , and  $H_t$  are predetermined and all other variables are non-predetermined.

Equations (A.1)-(A.2) follow from the definitions of  $A_{t+1}$  and  $H_{t+1}$ . Equation (A.3) defines the capital holding of young entrepreneurs at  $t$ , where  $M_t$  given in (A.12) is the total initial endowment of young entrepreneurs derived from (3). Equation (A.4) is the no-arbitrage condition. Equations (A.5)-(A.8) are the firm's first-order conditions with respect to  $k_t$ ,  $n_{c,t}$ ,  $n_{h,t}$ , and  $l_t$ , respectively. Equation (A.9) is the labor market clearing condition. Equations (A.10) and (A.11) follow from the definitions of  $Y_t$  and  $Y_{h,t}$ .

Our proofs actually rely on a two-variable system, simplified from the system (A.1)-(A.12). We discuss this simplified system next.

## A. 2 Dynamics of $(H_t, \frac{K_t}{Q_{t-1}})$ in a Simplified System

First, we show that  $N_{h,t}$  can be written as a decreasing function of  $\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}$ , which is useful when we study the equilibrium dynamics below.

**Lemma 1** *In any equilibrium,  $N_{h,t} = f\left(\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}\right)$  for  $t \geq 1$ , where  $f(\cdot)$  is a fixed strictly decreasing function.*

**Proof:** Equations (A.6) and (A.7) imply

$$(1 - \alpha)(1 - \tau) \left[ A_t / (K_t^\rho N_{c,t}^{1-\rho}) \right]^\theta K_t^\alpha N_{c,t}^{-\alpha} = (1 - \alpha_l) Q_t L_t^{\alpha_l} N_{h,t}^{-\alpha_l}. \quad (\text{A.13})$$

Substituting (A.5) into (A.13) and simplifying the latter equation, we have

$$\frac{N_{h,t}^{\alpha_l}}{1 - N_{h,t}} = \frac{(1 - \alpha_l)\alpha(1 - \psi)}{(1 - \alpha) \frac{R_t K_t}{Q_t L_t^{\alpha_l}}} = \frac{(1 - \alpha_l)\alpha(1 - \psi)}{(1 - \alpha)(1 - \delta_h) \frac{K_t}{Q_{t-1} L_t^{\alpha_l}}}, \quad (\text{A.14})$$

where the second equality uses the no-arbitrage condition (A.4). Because  $\frac{N_{h,t}^{\alpha_l}}{1 - N_{h,t}}$  is strictly increasing in  $N_{h,t} \in (0, 1)$ , the above equation defines  $N_{h,t} \in (0, 1)$  as a strictly decreasing function of  $\frac{K_t}{Q_{t-1} L_t^{\alpha_l}}$ . ■

Second, we show that the equilibrium dynamics of  $(H_t, \frac{K_t}{Q_{t-1}})$  satisfy a system of two difference equations for  $t \geq 1$ :

$$H_{t+1} = (1 - \delta_h)H_t + L_t^\alpha f\left(\frac{K_t}{Q_{t-1}L_t^{\alpha_l}}\right)^{1-\alpha_l}, \quad (\text{A.15})$$

$$\frac{K_{t+1}}{Q_t} = \frac{1}{z} \frac{K_t}{Q_{t-1}} - H_{t+1}, \quad (\text{A.16})$$

where  $z$  is defined in (12) and  $f(\cdot)$  is from Lemma 1.

Since (A.15) follows directly from  $H_{t+1} = (1 - \delta_h)H_t + Y_{h,t}$  and Lemma 1, we shall focus our discussion on (A.16). The initial wealth  $m_{t+1}$  of an entrepreneur born in  $t + 1$  satisfies

$$m_{t+1} = \alpha(1 - \tau)\psi \hat{A}_{t+1}^\theta K_{t+1}^\alpha N_{h,t+1}^{1-\alpha} = \frac{\psi}{(1 - \psi)} R_{t+1} K_{t+1},$$

where the second equality uses (A.5). Therefore,

$$\begin{aligned} \frac{K_{t+1}}{Q_t} + H_{t+1} &= \frac{K_{t+1} + Q_t H_{t+1}}{Q_t} = \frac{\beta}{(1 + \beta)} \frac{m_t}{Q_t} \\ &= \frac{\psi\beta}{(1 - \psi)(1 + \beta)} \frac{R_t K_t}{Q_t} = \frac{(1 - \delta_h)\psi\beta}{(1 - \psi)(1 + \beta)} \frac{K_t}{Q_{t-1}}, \end{aligned}$$

where the second equality uses (1) and the last equality follows from the no-arbitrage condition (A.4).

In period  $t = 0$ , we have

$$R_0 = \alpha(1 - \tau)(1 - \psi) \left[ A_0 / (K_0^\rho N_{c,0}^{1-\rho}) \right]^\theta K_0^{\alpha-1} N_{c,0}^{1-\alpha}.$$

and the last equality in (A.14) does not hold because (A.4) does not hold for  $R_0$ . Using (A.13) for  $t = 0$  and  $N_{c,0} + N_{h,0} = 1$ , we can show that  $Q_0$  is a function of  $N_{h,0}$ .

## Proof of Proposition 1

First, we show (13). In a bubbly steady state, equations (A.15)-(A.16) become

$$H^b = (1 - \delta_h)H^b + (L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l}, \quad (\text{A.17})$$

$$\frac{K^b}{Q^b} + H^b = \frac{K^b}{Q^b}/z, \quad (\text{A.18})$$

where  $N_h^b = f\left(\frac{K^b}{Q^b(L^*)^{\alpha_l}}\right)$ . It follows from (A.18) that  $H^b = (1/z - 1)\frac{K^b}{Q^b}$ , which implies

$$\phi^b = \frac{H^b Q^b}{H^b Q^b + K^b} = 1 - z.$$

Second, we show (14). It follows from the definition of  $f(\cdot)$  in Lemma 1 that

$$\frac{(N_h^b)^{\alpha_l}}{N_c^b} = \frac{(1 - \alpha_l)\alpha(1 - \psi)}{(1 - \alpha)(1 - \delta_h)\frac{K^b}{Q^b(L^*)^{\alpha_l}}},$$

which is rewritten as

$$\frac{K^b}{Q^b(L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l}} \frac{1 - N_c^b}{N_c^b} = \frac{(1 - \alpha_l)\alpha(1 - \psi)}{(1 - \alpha)(1 - \delta_h)}.$$

Substituting

$$\frac{K^b}{Q^b} = \frac{z}{1 - z}H^b = \frac{z}{(1 - z)\delta_h}(L^*)^{\alpha_l}(N_h^b)^{1-\alpha_l}$$

into the above equation, we have

$$\frac{z}{(1 - z)\delta_h} \frac{1 - N_c^b}{N_c^b} = \frac{(1 - \alpha_l)\alpha(1 - \psi)}{(1 - \alpha)(1 - \delta_h)},$$

which implies (14).

Third, we can derive that

$$p_L^b L^* = \alpha_l Q^b (L^*)^{\alpha_l} (N_h^b)^{1-\alpha_l} = \alpha_l \delta_h Q^b H^b = \alpha_l \delta_h \phi^b \frac{\beta}{1 + \beta} \alpha \psi (1 - \tau) Y^b,$$

where the first equality follows from (A.8), the second from (A.17), and the last from (4). Then equation (15) follows from the above equation and (5).

Finally, equation (16) follows from (4). Q.E.D.

## Proof of Proposition 2

We will show that, for any given  $(K_0, A_0, H_0)$ , there exists a unique  $Q_0 > 0$  such that the system (A.1)-(A.12) starting from  $(K_0, A_0, H_0, Q_0)$  converges to a bubbly steady state.<sup>13</sup> For simplicity, we first focus on the simplified system (A.15)-(A.16) for  $(H_t, \frac{K_t}{Q_{t-1}})$ , and extend this system to period

<sup>13</sup>At the beginning of period 0, if  $(K_0, A_0, H_0, Q_0)$  are known, then  $(R_0, w_0, p_{L0}, N_{10}, N_{20}, Y_0, Y_{h0}, M_0)$  can be solved as functions of  $(K_0, A_0, H_0, Q_0)$  from (A.5)-(A.12).

$t = 0$  by introducing a variable  $Q_{-1}$ . We show that there exists a unique  $Q_{-1}$  starting from which the system (A.15)-(A.16) converges.

The proof consists of three steps. Step 1 discusses initial conditions from which the system (A.15)-(A.16) will diverge. Step 2 shows that there exists a unique  $Q_{-1}$  from which the system (A.15)-(A.16) converges to a steady state. Step 3 shows that this unique  $Q_{-1}$  implies a unique  $Q_0$ .

**Step 1.** To simplify notation, we denote  $\frac{K_t}{Q_{t-1}}$  by  $X_t$  in the following proof. We introduce two sets of initial conditions from which the system (A.15)-(A.16) will eventually diverge. In particular, define sets  $\mathcal{U}_t$  and  $\mathcal{L}_t$  as follows:

$$\begin{aligned}\mathcal{U}_t &\equiv \{(H, X) : H'(H, X, \bar{L}_t) < H^*(\underline{L}_t) \text{ and } X'(H, X, \bar{L}_t) > X^*(\underline{L}_t)\}, \\ \mathcal{L}_t &\equiv \{(H, X) : H'(H, X, \underline{L}_t) > H^*(\bar{L}_t) \text{ and } X'(H, X, \underline{L}_t) < X^*(\bar{L}_t)\},\end{aligned}$$

where  $(H^*(L), X^*(L))$  denote the steady state when the land supply is always equal to  $L$ , and  $\underline{L}_t \equiv \inf_{s \geq t} L_s$ ,  $\bar{L}_t \equiv \sup_{s \geq t} L_s$ . Here  $H'$  and  $X'$  represent the right-hand sides of (A.15) and (A.16),

$$\begin{aligned}H'(H, X, L) &\equiv (1 - \delta_h)H + L^{\alpha_l} f\left(\frac{X}{L^{\alpha_l}}\right)^{1-\alpha_l}, \\ X'(H, X, L) &\equiv \frac{1}{z}X - (1 - \delta_h)H - L^{\alpha_l} f\left(\frac{X}{L^{\alpha_l}}\right)^{1-\alpha_l}.\end{aligned}$$

The divergence of  $\mathcal{U}_t$  and  $\mathcal{L}_t$  is verified in the following lemma.

**Lemma 2** *If  $(H_t, X_t) \in \mathcal{U}_t$ , then  $\lim_{s \rightarrow \infty} X_s = \infty$ . If  $(H_t, X_t) \in \mathcal{L}_t$ , then  $X_s < 0$  for finite  $s$ .*

**Proof:** Suppose  $(H_t, X_t) \in \mathcal{U}_t$ . Because  $H'$  is increasing in  $L$  and  $X'$  is decreasing in  $L$ ,

$$\begin{aligned}H_{t+1} &= H'(H_t, X_t, L_t) \leq H'(H_t, X_t, \bar{L}_t) < H^*(\underline{L}_t), \\ X_{t+1} &= X'(H_t, X_t, L_t) \geq X'(H_t, X_t, \bar{L}_t) > X^*(\underline{L}_t).\end{aligned}$$

By induction, we can show that  $H_s < H^*(\underline{L}_t)$  and  $X_s > X^*(\underline{L}_t)$  for all  $s \geq t + 1$ . It follows from (A.16) that for all  $s \geq t + 1$ ,

$$X_{s+1} - X^*(\underline{L}_t) = \frac{X_s - X^*(\underline{L}_t)}{z} - H_{s+1} + H^*(\underline{L}_t) > \frac{X_s - X^*(\underline{L}_t)}{z},$$

which implies  $\lim_{s \rightarrow \infty} X_s = \infty$  since  $z < 1$ . Similarly, if  $(X_t, H_t) \in \mathcal{L}_t$ , then by induction we can show  $X_s < X^*(\bar{L}_t)$  and  $H_s > H^*(\bar{L}_t)$  for all  $s \geq t + 1$ . Moreover,

$$X_{s+1} - X^*(\bar{L}_t) < \frac{X_s - X^*(\bar{L}_t)}{z},$$

which implies  $X_s < 0$  for finite  $s$ . ■

The following alternative definitions of  $\mathcal{U}_t$  and  $\mathcal{L}_t$  are used in the proof of Lemma 3 below. The conditions  $H'(H, X, \bar{L}_t) < H^*(\underline{L}_t)$  and  $X'(H, X, \bar{L}_t) > X^*(\underline{L}_t)$  are, respectively,

$$H < \frac{H^*(\underline{L}_t) - \bar{L}_t^{\alpha_l} f\left(\frac{X}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h}, \quad H < \frac{X/z - X^*(\underline{L}_t) - \bar{L}_t^{\alpha_l} f\left(\frac{X}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h}.$$

Therefore,

$$\mathcal{U}_t \equiv \left\{ (H, X) : H < \frac{\min\{H^*(\underline{L}_t), X/z - X^*(\underline{L}_t)\} - \bar{L}_t^{\alpha_l} f\left(\frac{X}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \right\}.$$

Similarly,

$$\mathcal{L}_t \equiv \left\{ (H, X) : H > \frac{\max\{H^*(\bar{L}_t), X/z - X^*(\bar{L}_t)\} - \underline{L}_t^{\alpha_l} f\left(\frac{X}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \right\}.$$

**Step 2.** We show a unique  $X_0$  from which the system converges. Above  $X_0$ , the system enters  $\mathcal{U}_t$  eventually. Below  $X_0$ , the system enters  $\mathcal{L}_t$  eventually.

**Lemma 3** *For any  $H_0 > 0$ , there exists a unique  $X_0$  such that the system starting from  $(H_0, X_0)$  converges to  $(H^*(L^*), X^*(L^*))$ , where  $L^*$  is the land supply in the long run. The system starting from  $\tilde{X}_0 > X_0$  satisfies  $\lim_{t \rightarrow \infty} \tilde{X}_t = \infty$ , and that from  $\tilde{X}_0 < X_0$  satisfies  $\tilde{X}_t < 0$  for some  $t > 0$ .*

**Proof:** For any  $H_0 > 0$ , define sets  $A$  and  $B$  as follows.

$$A \equiv \{X_0 : \text{the system starting from } (H_0, X_0) \text{ satisfies } \lim_{t \rightarrow \infty} X_t = \infty\},$$

$$B \equiv \{X_0 : \text{the system starting from } (H_0, X_0) \text{ satisfies } X_t < 0 \text{ for some } t\}.$$

First, both  $A$  and  $B$  are nonempty but  $A \cap B = \emptyset$ .  $B \neq \emptyset$  because if  $X_0$  is sufficiently small then  $X_1 = X_0/z - H_1 < X_0/z - (1 - \delta_h)H_0 < 0$ . To prove  $A \neq \emptyset$ , pick a sufficiently large  $Y$  such that  $\bar{L}_0^{\alpha_l} f\left(\frac{Y}{\bar{L}_0^{\alpha_l}}\right)^{1-\alpha_l} < \delta H_0$ . We show that  $\lim_t X_t = \infty$  if  $X_0 > \max\{Y, \frac{2H_0}{1/z-1}\}$ . To show this, it is sufficient to show  $H_t \leq H_0$  and  $X_t > \frac{1/z+1}{2}X_{t-1}$  for all  $t \geq 1$ . If  $t = 1$ , then

$$H_1 = (1 - \delta_h)H_0 + L_0^{\alpha_l} f\left(\frac{X_0}{L_0^{\alpha_l}}\right)^{1-\alpha_l} < (1 - \delta_h)H_0 + L_0^{\alpha_l} f\left(\frac{Y}{L_0^{\alpha_l}}\right)^{1-\alpha_l} < H_0,$$

$$X_1 = X_0/z - H_1 > X_0/z - H_0 > X_0/z - X_0 \frac{1/z-1}{2} = \frac{1/z+1}{2}X_0.$$

By induction, suppose  $H_s \leq H_0$  and  $X_s > \frac{1/z+1}{2}X_{s-1}$  for all  $s \leq t$ , then for  $t+1$ ,

$$H_{t+1} = (1 - \delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < (1 - \delta_h)H_0 + L_t^{\alpha_l} f\left(\frac{Y}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < H_0,$$

$$X_{t+1} = X_t/z - H_{t+1} > X_t/z - H_0 > X_t/z - X_t \frac{1/z-1}{2} = \frac{1/z+1}{2}X_t.$$

$A \cap B = \emptyset$  because the system is terminated after  $X_t$  reaches negative values. So  $X_t$  cannot converge to  $\infty$  at the same time.

Second, both  $A$  and  $B$  are open.  $B$  is open because if  $X_t < 0$  for some finite  $t$ , then continuity implies that  $X_t$  remains negative if there is a small change to  $X_0$ .  $A$  is open because if  $\lim_{t \rightarrow \infty} X_t = \infty$ , then equation (A.15) and  $f(\infty) = 0$  imply  $\lim_{t \rightarrow \infty} H_t = 0$ . Therefore,  $H_t < \frac{\min\{H^*(\underline{L}_t), X_t/z - X^*(\underline{L}_t)\} - \bar{L}_t^{\alpha_l} f\left(\frac{X_t}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h}$  for large  $t$ . It follows from  $(H_t, X_t) \in \mathcal{U}_t$  and Lemma 2 that  $\lim_{t \rightarrow \infty} X_t = \infty$ .

Third,  $(0, \infty) \setminus (A \cup B)$  is nonempty because  $(0, \infty)$  is a connected set. Pick  $X_0 \in (0, \infty) \setminus (A \cup B)$  and we show below that the system starting from  $(H_0, X_0)$  converges to  $(H^*(L^*), X^*(L^*))$ , that is, for any  $\epsilon > 0$ , there exists a  $N$  such that  $(H_t, X_t) \in (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon) \times (X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$  for all  $t \geq N$ . We shall repeatedly use the fact that  $(H_t, X_t) \notin \mathcal{L}_t \cup \mathcal{U}_t$  for all  $t$ .

- (i) We show that there exists a small  $\epsilon_2 \in (0, \epsilon)$  such that  $t > \frac{1}{\epsilon_2}$  and  $H_t \in (H^*(L^*) - \epsilon_2, H^*(L^*) + \epsilon_2)$  imply  $(H_{t+1}, X_{t+1}) \in (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon) \times (X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$ . Because  $H'(H, X, L)$  and  $X'(H, X, L)$  are continuous functions, there exists  $\epsilon_3 > 0$  such that  $(H', X') \in (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon) \times (X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$  for all  $(H, X, L) \in (H^*(L^*) - \epsilon_3, H^*(L^*) + \epsilon_3) \times (X^*(L^*) - \epsilon_3, X^*(L^*) + \epsilon_3) \times (L^* - \epsilon_3, L^* + \epsilon_3)$ . We can choose a sufficiently small  $\epsilon_2 < \epsilon_3$  such that  $L_t \in (L^* - \epsilon_3, L^* + \epsilon_3)$  for all  $t > \frac{1}{\epsilon_2}$ . Because both  $\partial \mathcal{L}_t$  and  $\partial \mathcal{U}_t$  are upward sloping and continuous, we can also choose a sufficiently small  $\epsilon_2$  such that  $H_t \in (H^*(L^*) - \epsilon_2, H^*(L^*) + \epsilon_2)$  and  $(H_t, X_t) \notin \mathcal{L}_t \cup \mathcal{U}_t$  imply that  $(H_t, X_t) \in (H^*(L^*) - \epsilon_3, H^*(L^*) + \epsilon_3) \times (X^*(L^*) - \epsilon_3, X^*(L^*) + \epsilon_3)$  for all  $t > \frac{1}{\epsilon_2}$ .
- (ii) We show that there exists a small  $\epsilon_4 > 0$  such that  $t > \frac{1}{\epsilon_4}$  and  $H_t \leq H^*(L^*) - \epsilon_2$  imply  $H_{t+1} \in (H_t, H^*(L^*) + \epsilon_2)$ . To show  $H_{t+1} > H_t$ , choose a sufficiently small  $\epsilon_4$  such that for  $t \geq \frac{1}{\epsilon_4}$ ,

$$(a) \quad \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} > \delta(H^*(L^*) - \epsilon_2);$$

$$(b) \quad \frac{\min\{H^*(\underline{L}_t), X^*(L^*)/z - X^*(\underline{L}_t)\} - \bar{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} > H^*(L^*) - \epsilon_2.$$

Then  $(H_t, X_t) \notin \mathcal{U}_t$  implies that

$$\frac{\min\{H^*(\underline{L}_t), X_t/z - X^*(\underline{L}_t)\} - \bar{L}_t^{\alpha_l} f\left(\frac{X_t}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1-\delta_h} \leq H_t \leq H^*(L^*) - \epsilon_2,$$

which, together with (b), implies  $X_t < X^*(L^*)$ . It follows from (a) that

$$\underline{L}_t^{\alpha_l} f\left(\frac{X_t}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} \geq \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l} > \delta(H^*(L^*) - \epsilon_2) \geq \delta H_t.$$



Therefore,  $H_{t+1} = (1 - \delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} > H_t$ . To show  $H_{t+1} < H^*(L^*) + \epsilon_2$ , choose a sufficiently small  $\epsilon_4$  such that for  $t \geq \frac{1}{\epsilon_4}$ ,

$$\frac{\max\{H^*(\bar{L}_t), X^*(L^*)/z - X^*(\bar{L}_t)\} - \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} < H^*(L^*) + \epsilon_2.$$

By contradiction, suppose  $H_{t+1} \geq H^*(L^*) + \epsilon_2$ . Then  $X_{t+1} = X_t/z - H_{t+1} \leq X^*(L^*)/z - H^*(L^*) - \epsilon_2 < X^*(L^*)$ , and

$$\begin{aligned} & \frac{\max\{H^*(\bar{L}_{t+1}), X_{t+1}/z - X^*(\bar{L}_{t+1})\} - \underline{L}_{t+1}^{\alpha_l} f\left(\frac{X_{t+1}}{\underline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \\ & \leq \frac{\max\{H^*(\bar{L}_{t+1}), X^*(L^*)/z - X^*(\bar{L}_{t+1})\} - \underline{L}_{t+1}^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \\ & < H^*(L^*) + \epsilon_2 \leq H_{t+1}, \end{aligned}$$

which contradicts the fact that  $(H_{t+1}, X_{t+1}) \notin \mathcal{L}_t$ .

(iii) Symmetrically, we show that there exists a small  $\epsilon_4 > 0$  such that  $t > \frac{1}{\epsilon_4}$  and  $H_t \geq H^*(L^*) + \epsilon_2$  imply  $H_{t+1} \in (H^*(L^*) - \epsilon_2, H_t)$ . To show  $H_{t+1} < H_t$ , choose a sufficiently small  $\epsilon_4$  such that for  $t \geq \frac{1}{\epsilon_4}$ ,

$$\begin{aligned} \text{(a)} \quad & \bar{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l} < \delta(H^*(L^*) + \epsilon_2); \\ \text{(b)} \quad & \frac{\max\{H^*(\bar{L}_t), X^*(L^*)/z - X^*(\bar{L}_t)\} - \underline{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} < H^*(L^*) + \epsilon_2. \end{aligned}$$

Then  $(H_t, X_t) \notin \mathcal{L}_t$  implies that

$$\frac{\max\{H^*(\bar{L}_t), X_t/z - X^*(\bar{L}_t)\} - \underline{L}_t^{\alpha_l} f\left(\frac{X_t}{\underline{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \geq H_t \geq H^*(L^*) + \epsilon_2,$$

which, together with (b), implies  $X_t > X^*(L^*)$ . It follows from (a) that

$$L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} \leq \bar{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l} < \delta(H^*(L^*) + \epsilon_2) \leq \delta H_t.$$

Therefore,  $H_{t+1} = (1 - \delta_h)H_t + L_t^{\alpha_l} f\left(\frac{X_t}{L_t^{\alpha_l}}\right)^{1-\alpha_l} < H_t$ . To show  $H_{t+1} > H^*(L^*) - \epsilon_2$ , choose a sufficiently small  $\epsilon_4$  such that for  $t \geq \frac{1}{\epsilon_4}$ ,

$$\frac{\min\{H^*(\underline{L}_t), X^*(L^*)/z - X^*(\underline{L}_t)\} - \bar{L}_t^{\alpha_l} f\left(\frac{X^*(L^*)}{\bar{L}_t^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} > H^*(L^*) - \epsilon_2.$$

By contradiction, suppose  $H_{t+1} \leq H^*(L^*) - \epsilon_2$ . Then  $X_{t+1} = X_t/z - H_{t+1} \geq X^*(L^*)/z - H^*(L^*) + \epsilon_2 > X^*(L^*)$ , and

$$\begin{aligned} & \frac{\min \{H^*(L_{t+1}), X_{t+1}/z - X^*(L_{t+1})\} - \bar{L}_{t+1}^{\alpha_l} f\left(\frac{X_{t+1}}{\bar{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \\ & \geq \frac{\min \{H^*(L_{t+1}), X^*(L^*)/z - X^*(L_{t+1})\} - \bar{L}_{t+1}^{\alpha_l} f\left(\frac{X^*(L^*)}{\bar{L}_{t+1}^{\alpha_l}}\right)^{1-\alpha_l}}{1 - \delta_h} \\ & > H^*(L^*) - \epsilon_2 \geq H_{t+1}, \end{aligned}$$

which contradicts the fact that  $(H_{t+1}, X_{t+1}) \notin \mathcal{U}_t$ .

- (iv) We show that set  $(H^*(L^*) - \epsilon, H^*(L^*) + \epsilon)$  is absorbing for  $t \geq 1/\epsilon_4$ . Starting from  $(X^*(L^*) - \epsilon, X^*(L^*) - \epsilon_2) \cup [X^*(L^*) + \epsilon_2, X^*(L^*) + \epsilon)$ , the path monotonically converges to  $(X^*(L^*) - \epsilon_2, X^*(L^*) + \epsilon_2)$ ; starting from  $(X^*(L^*) - \epsilon_2, X^*(L^*) + \epsilon_2)$ , the path stays in  $(X^*(L^*) - \epsilon, X^*(L^*) + \epsilon)$ . Even if  $H_0 \notin (H^*(L^*) - \epsilon, H^*(L^*) + \epsilon)$ , the path will enter  $(H^*(L^*) - \epsilon, H^*(L^*) + \epsilon)$  and stay in it forever.

Fourth, if  $\tilde{X}_0 > X_0$ , then by induction we can show that  $\tilde{H}_t < H_t$  and  $\tilde{X}_t > X_t$  for all  $t \geq 1$ . Therefore,

$$\tilde{X}_{t+1} - X_{t+1} = (\tilde{X}_t - X_t)/z - (\tilde{H}_{t+1} - X_{t+1}) > (\tilde{X}_t - X_t)/z,$$

which implies  $\lim_t (\tilde{X}_t - X_t) = \infty$ . Therefore,  $\lim_t \tilde{X}_t = \infty + X^*(L^*) = \infty$ . Similarly, if  $\tilde{X}_0 < X_0$ , then we can show that  $\tilde{H}_t > H_t$  and  $\tilde{X}_t < X_t$  for all  $t \geq 1$ . Therefore,  $\lim_t \tilde{X}_t = -\infty$ . This step also implies that  $X_0$  is unique.  $\blacksquare$

**Step 3.** We show that for any  $(A_0, H_0, K_0)$ , there exists a unique  $Q_0$  such that the system (A.1)-(A.12) starting from  $(K_0, A_0, H_0, Q_0)$  converges to  $(H^*(L^*), X^*(L^*))$ . Lemma 3 shows that, given  $H_0$ , there exists a unique  $\frac{K_0}{Q_{-1}}$  such that the system converges to  $(H^*(L^*), X^*(L^*))$ . There is an increasing and one-to-one mapping between  $Q_{-1}$  and  $Q_0$ . Equation (A.13) implies

$$Q_0 = \frac{(1 - \alpha)(1 - \tau)A_0^\theta K_0^{\alpha - \theta\rho} (1 - N_{h,0})^{-\alpha - \theta(1-\rho)} N_{h,0}^{\alpha_l}}{(1 - \alpha_l)L_0^{\alpha_l}}.$$

Substituting  $N_{h,0} = f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right)$  into the above yields

$$Q_0 = \frac{(1 - \alpha)(1 - \tau)A_0^\theta K_0^{\alpha - \theta\rho} \left(1 - f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right)\right)^{-\alpha - \theta(1-\rho)} f\left(\frac{K_0}{Q_{-1}L_0^{\alpha_l}}\right)^{\alpha_l}}{(1 - \alpha_l)L_0^{\alpha_l}},$$

which is increasing in  $Q_{-1}$ .

Suppose  $Q_0$  corresponds to the unique  $Q_{-1}$  in Lemma 3 from which the system  $(H_t, X_t)$  converges to a steady state. If  $\tilde{Q}_0 < Q_0$ , then  $\tilde{Q}_{-1} < Q_{-1}$ , which implies  $\tilde{X}_0 = \frac{K_0}{\tilde{Q}_{-1}} > \frac{K_0}{Q_{-1}} = X_0$  and

$\lim_t \tilde{X}_t = \infty$ . This represents an equilibrium in which  $\lim_t \tilde{Q}_t = 0$ . However, if  $\tilde{Q}_0 > Q_0$ , then  $\tilde{Q}_{-1} > Q_{-1}$  and  $\tilde{X}_t < 0$  for some  $t$ . Equilibrium does not exist in the latter case.

### Proof of Proposition 3

If  $A$  and  $K$  are both linear functions of  $Y$ ,

$$A = \lambda_A Y, \quad (\text{A.19})$$

$$K = \lambda_K Y, \quad (\text{A.20})$$

it follows from (A.19), (A.20) and the production function  $Y = A^\theta K^{\alpha-\rho\theta} N_c^{1-\alpha-(1-\rho)\theta}$  that

$$Y = \left( \lambda_A^\theta \lambda_K^{\alpha-\rho\theta} N_c^{1-\alpha-(1-\rho)\theta} \right)^{\frac{1}{1-\alpha-(1-\rho)\theta}}.$$

Therefore, (9), (10) and  $N_c^n = 1$  in the bubbleless steady state imply

$$Y^n = \left( (\delta_a^{-1}\tau)^\theta \left( \frac{\beta}{1+\beta} \alpha\psi(1-\tau) \right)^{\alpha-\rho\theta} \right)^{\frac{1}{1-\alpha-(1-\rho)\theta}},$$

while (15) and (16) in the bubbly steady state imply

$$Y^b = \left( (\delta_a^{-1}\alpha_l\delta_h\phi^b \frac{\beta}{1+\beta} \alpha\psi(1-\tau) + \delta_a^{-1}\tau)^\theta \left( (1-\phi^b) \frac{\beta}{1+\beta} \alpha\psi(1-\tau) \right)^{\alpha-\rho\theta} (N_c^b)^{1-\alpha-(1-\rho)\theta} \right)^{\frac{1}{1-\alpha-(1-\rho)\theta}}.$$

These calculations show that  $Y^b > Y^n$  if and only if

$$\left( \frac{Y^b}{Y^n} \right)^{1-\alpha-(1-\rho)\theta} = \left( 1 + \frac{\alpha_l\delta_h\phi^b\beta\alpha\psi(1-\tau)}{(1+\beta)\tau} \right)^\theta (1-\phi^b)^{\alpha-\rho\theta} (N_c^b)^{1-\alpha-(1-\rho)\theta} > 1.$$

The left-hand side of the above inequality is increasing in  $\theta$  because  $1 + \frac{\alpha_l\delta_h\phi^b\beta\alpha\psi(1-\tau)}{(1+\beta)\tau} > 1$  and  $N_c^b < 1$ . The other two equations in the proposition follow from equations (10), (16), (9), and (15).

## B The Extended Model in Section 4

### B.1 Decision Rules

We show two properties of the optimal consumption-savings allocation in the entrepreneurs' problem. First, an age  $j \geq 2$  entrepreneur's consumption  $c_{j,t}^e$  in period  $t$  satisfies

$$c_{j,t}^e = \frac{1-\beta}{1-\beta^{T-(j-1)}} \left( R_t k_{j,t} + (Q_t(1-\delta_h) + r_t) h_{j,t} + R^f b_{j,t}^e \right). \quad (\text{B.1})$$

The newborn age 1 entrepreneur's wealth is inherited from his/her parents. Then his/her consumption is given by

$$c_{1,t}^e = \frac{1-\beta}{1-\beta^T} m_t. \quad (\text{B.2})$$

Thus, consumption is a fixed fraction of an entrepreneur's wealth in any period. This fraction depends on the entrepreneur's age, but not on his/her wealth level or the uncertainty in the rate of return from capital and housing.

Second, all entrepreneurs make the same portfolio choice at time  $t$  regardless of age, that is,  $h_{j+1,t+1}/k_{j+1,t+1}$  is independent of  $j$ . These properties will simplify the calculation of equilibrium allocations in the extended model.

The following two lemmas prove these properties. We need to use an age- $j$  entrepreneur's budget constraints at time  $t$ , which are explicitly written as

$$\begin{aligned} & Q_t h_{j+1,t+1} + k_{j+1,t+1} + c_{j,t}^e + b_{j+1,t+1}^e \\ = & \begin{cases} m_t, & j = 1, \\ R_t k_{j,t} + (Q_t(1 - \delta_h) + r_t) h_{j,t} + R^f b_{j,t}^e, & 2 \leq j \leq T. \end{cases} \end{aligned} \quad (\text{B.3})$$

**Lemma 4** *With logarithmic utility and  $R_{t+1} > R^f$ , we have  $b_{j+1,t+1}^e = -\xi k_{j+1,t+1}$  and equations (B.1) and (B.2) hold.*

**Proof:** We prove (B.1) by backward induction in  $j$ . If  $j = T$ , then  $\frac{1-\beta}{1-\beta^{T-(j-1)}} = 1$  and an entrepreneur in his/her last period of lifespan should obviously consume all the wealth.

Suppose (B.1) holds for  $j + 1$ , i.e.,

$$\begin{aligned} c_{j+1,t+1}^e &= \frac{1-\beta}{1-\beta^{T-j}} \left( R_{t+1} k_{j+1,t+1} + (Q_{t+1}(1 - \delta_h) + r_{t+1}) h_{j+1,t+1} + R^f b_{j+1,t+1}^e \right) \\ &= \frac{1-\beta}{1-\beta^{T-j}} \left( R_t k_{j,t} + (Q_t(1 - \delta_h) + r_t) h_{j,t} + R^f b_{j,t}^e - c_{j,t}^e \right) \times \\ &\quad (\tilde{R}_{t+1}(1 - \phi_{j,t}) + R_{t+1}^h \phi_{j,t}), \end{aligned}$$

where we have used (B.3),

$$\begin{aligned} \phi_{j,t} &= \frac{Q_t h_{j+1,t+1}}{k_{j+1,t+1} + Q_t h_{j+1,t+1} + b_{j+1,t+1}^e} = \frac{Q_t h_{j+1,t+1}}{(1 - \xi) k_{j+1,t+1} + Q_t h_{j+1,t+1}}, \\ \tilde{R}_{t+1} &\equiv \frac{R_{t+1} - \xi R^f}{1 - \xi}, \quad R_{t+1}^h \equiv \frac{Q_{t+1}(1 - \delta_h) + r_{t+1}}{Q_t}, \end{aligned}$$

and  $b_{j+1,t+1}^e = -\xi k_{j+1,t+1}$ . Notice that the borrowing constraint always binds when  $R_{t+1} > R^f$ . Substituting the above consumption equation into the entrepreneur's Euler equation,

$$u'(c_{j,t}^e) = \beta E_t[u'(c_{j+1,t+1}^e)(\tilde{R}_{t+1}(1 - \phi_{j,t}) + R_{t+1}^h \phi_{j,t})],$$

we have

$$\frac{1}{c_{j,t}^e} = \beta E_t \left[ \frac{1}{\frac{1-\beta}{1-\beta^{T-j}} \left( R_t k_{j,t} + (Q_t(1 - \delta_h) + r_t) h_{j,t} + R^f b_{j,t}^e - c_{j,t}^e \right)} \right].$$

Solving the above equation for  $c_{j,t}^e$  yields (B.1)

Finally for  $j = 1$ , the entrepreneur's wealth is  $m_t$ . We then obtain (B.2). ■

**Lemma 5** *With logarithmic utility and  $R_{t+1} > R^f$ , all entrepreneurs make the same portfolio choice at time  $t$  regardless of age, i.e.,  $\phi_{j,t}$  is independent of  $j$ .*

**Proof:** The no-arbitrage condition is

$$(1 - p_{t+1}) \frac{u'(c_{j,t+1}^{e+})}{u'(c_{j,t+1}^{e-})} R_{t+1}^{h+} + p_{t+1} R_{t+1}^{h-} = (1 - p_{t+1}) \frac{u'(c_{j,t+1}^{e+})}{u'(c_{j,t+1}^{e-})} \tilde{R}_{t+1}^+ + p_{t+1} \tilde{R}_{t+1}^-.$$

It follows from Lemma 4 that  $\frac{u'(c_{j,t+1}^{e+})}{u'(c_{j,t+1}^{e-})} = \frac{c_{j,t+1}^{e-}}{c_{j,t+1}^{e+}}$  is equal to the wealth ratio between state  $-$  and state  $+$ . Therefore

$$\begin{aligned} & (1 - p_{t+1}) \frac{(R_{t+1}^{h-} \phi_{j,t} + \tilde{R}_{t+1}^-(1 - \phi_{j,t}))}{(R_{t+1}^{h+} \phi_{j,t} + \tilde{R}_{t+1}^+(1 - \phi_{j,t}))} R_{t+1}^{h+} + p_{t+1} R_{t+1}^{h-} \\ &= (1 - p_{t+1}) \frac{(R_{t+1}^{h-} \phi_{j,t} + \tilde{R}_{t+1}^-(1 - \phi_{j,t}))}{(R_{t+1}^{h+} \phi_{j,t} + \tilde{R}_{t+1}^+(1 - \phi_{j,t}))} \tilde{R}_{t+1}^+ + p_{t+1} \tilde{R}_{t+1}^-. \end{aligned} \quad (\text{B.4})$$

From the above equation, we can solve  $\phi_{j,t}$  as

$$\phi_{j,t} = \frac{\tilde{R}_{t+1}^- \tilde{R}_{t+1}^+ - (1 - p_{t+1}) \tilde{R}_{t+1}^- R_{t+1}^{h+} - p_{t+1} R_{t+1}^{h-} \tilde{R}_{t+1}^+}{R_{t+1}^{h-} R_{t+1}^{h+} + \tilde{R}_{t+1}^- \tilde{R}_{t+1}^+ - R_{t+1}^{h-} \tilde{R}_{t+1}^+ - \tilde{R}_{t+1}^- R_{t+1}^{h+}}.$$

Because the right-hand side of the above does not depend on  $j$ , entrepreneurs of different ages make the same portfolio choice.  $\blacksquare$

Let  $(K_t(j), H_t(j))_{j=1}^T$  denote the aggregate holdings of capital and houses at the beginning of time  $t$  for ages  $j = 1, \dots, T$ . Because Lemma 5 implies that  $H_t(j) = \frac{K_t(j)}{K_t} H_t$ , we will include  $K_t$ ,  $H_t$ , and  $\{K_t(j)\}_{j=1}^T$  in our state variables but not  $\{H_t(j)\}_{j=1}^T$  since  $\{H_t(j)\}_{j=1}^T$  can be inferred from the others.

## B. 2 Bubbleless Equilibrium

The dynamic system for the bubbleless equilibrium in the extended model contains  $3T + 16$  variables  $B_t^g$ ,  $A_t$ ,  $H_t$ ,  $K_t(j)$ ,  $B_t^e(j)$ ,  $K_t$ ,  $Z_t$ ,  $Z_t(j)$ ,  $R_t$ ,  $w_t$ ,  $p_{L,t}$ ,  $K_{c,t}$ ,  $K_{h,t}$ ,  $N_{c,t}$ ,  $N_{h,t}$ ,  $Y_t$ ,  $Y_{h,t}$ ,  $M_t$ ,  $Q_t$ , for  $j = 1, \dots, T$ , that satisfy the following system of  $3T + 16$  difference equations for  $t \geq 0$ :

$$B_{t+1}^g = \xi_g p_{L,t} L_t, \quad (\text{B.5})$$

$$A_{t+1} = (1 - \delta_a) A_t - G_t - R^f B_t^g + \tau Y_t + \tau_h Q_t Y_{ht} + p_{L,t} L_t + B_{t+1}^g, \quad (\text{B.6})$$

$$H_{t+1} = (1 - \delta_h) H_t + Y_{ht}, \quad (\text{B.7})$$

$$K_{t+1} = \frac{1}{1 - \xi} (Z_t - Q_t H_{t+1}), \quad (\text{B.8})$$

$$K_t(1) = 0, \quad (\text{B.9})$$

$$K_{t+1}(j+1) = K_{t+1} \frac{Z_t(j)}{Z_t}, \quad \forall j = 1, \dots, T-1, \quad (\text{B.10})$$

$$B_{t+1}^e(j+1) = -\xi K_{t+1}(j+1), \quad \forall j = 0, 1, \dots, T-1, \quad (\text{B.11})$$

$$Z_t(1) = \left(1 - \frac{1-\beta}{1-\beta^T}\right) M_t, \quad (\text{B.12})$$

$$Z_t(j) = \left[1 - \frac{1-\beta}{1-\beta^{T-(j-1)}}\right] \times \left[K_t(1-\xi)\tilde{R}_t + H_t((1-\delta_h)Q_t + r_t)\right] \frac{K_t(j)}{K_t}, \quad \forall j = 2, \dots, T, \quad (\text{B.13})$$

$$Z_t = \sum_{j=1}^T Z_t(j), \quad (\text{B.14})$$

$$K_t = K_{c,t} + K_{h,t}, \quad (\text{B.15})$$

$$N_t = N_{c,t} + N_{h,t}, \quad (\text{B.16})$$

$$R_t = \alpha(1-\tau)(1-\psi)\hat{A}_t^\theta K_{c,t}^{\alpha-1} (e_t N_{c,t})^{1-\alpha} + (1-\delta_k), \quad (\text{B.17})$$

$$R_t = \alpha_k(1-\tau_h)(1-\psi)Q_t \times \left((1-gl)^t L_t\right)^{\alpha_l} K_{h,t}^{\alpha_k-1} (e_t N_{h,t})^{1-\alpha_l-\alpha_k} + (1-\delta_k), \quad (\text{B.18})$$

$$w_t = (1-\alpha)(1-\tau)\hat{A}_t^\theta K_{c,t}^\alpha e_t^{1-\alpha} N_{c,t}^{-\alpha}, \quad (\text{B.19})$$

$$w_t = (1-\alpha_l-\alpha_k)(1-\tau_h)Q_t \times \left((1-gl)^t L_t\right)^{\alpha_l} K_{h,t}^{\alpha_k} e_t^{1-\alpha_l-\alpha_k} N_{h,t}^{-\alpha_l-\alpha_k}, \quad (\text{B.20})$$

$$p_{Lt} = \alpha_l(1-\tau_h)Q_t(1-gl)^{t\alpha_l} L_t^{\alpha_l-1} K_{h,t}^{\alpha_k} (e_t N_{h,t})^{1-\alpha_l-\alpha_k}, \quad (\text{B.21})$$

$$Y_t = \hat{A}_t^\theta K_{c,t}^\alpha (e_t N_{c,t})^{1-\alpha}, \quad (\text{B.22})$$

$$Y_{h,t} = \left((1-gl)^t L_t\right)^{\alpha_l} K_{h,t}^{\alpha_k} (e_t N_{h,t})^{1-\alpha_l-\alpha_k}, \quad (\text{B.23})$$

$$M_t = \psi((1-\tau)\alpha Y_t + (1-\tau_h)\alpha_k Q_t Y_{h,t}), \quad (\text{B.24})$$

$$\tilde{R}_{t+1} = \frac{(1-\delta_h)Q_{t+1} + r_{t+1}}{Q_t}, \quad (\text{B.25})$$

where  $\hat{A}_t$  and  $\tilde{R}_t$  satisfy

$$\hat{A}_t = \frac{A_t}{K_{c,t}^\rho (e_t N_{c,t})^{1-\rho}}, \quad \tilde{R}_t = \frac{R_t - \xi R^f}{1-\xi}.$$

Workers' decision problem is much simpler. For our small open economy, their consumption/saving choices do not affect the above equilibrium system. Once obtaining a solution to the above system, we can derive the consumption rules for the entrepreneurs and workers. Here we omit the details.

Equations (B.5)-(B.7) follow from the definitions of  $B_{t+1}^g$ ,  $A_{t+1}$ , and  $H_{t+1}$ . Equation (B.8) computes the aggregate  $K_{t+1}$ , using the binding borrowing constraint of entrepreneurs. The variable  $Z_t$  denotes the aggregate wealth net of consumption across all entrepreneurs and  $Z_t(j)$  denotes the total after-consumption wealth of age- $j$  entrepreneurs. Equation (B.9) says that a newborn entrepreneur does not own capital. Equation (B.10) defines an age- $(j+1)$  entrepreneur's capital holding at  $t+1$ , whose age is  $j$  at period  $t$ . Here,  $\frac{K_{t+1}(j+1)}{Z_t(j)} = \frac{K_{t+1}}{Z_t}$  holds because Lemma 5 shows that

$$\frac{K_{t+1}(j+1)}{Z_t(j)} = \frac{k_{j+1,t+1}}{k_{j+1,t+1} + Q_t h_{j+1,t+1} + b_{j+1,t+1}^e} = \frac{1 - \phi_{j,t}}{1 - \xi}$$

is independent of  $j$ .

Equation (B.11) is the binding borrowing constraint of entrepreneurs. Equation (B.12) defines newly-born entrepreneurs' wealth  $Z_t(1)$  after consumption, where  $M_t$  given in (B.24) is their total initial endowment. Equation (B.13) defines total age- $j$  entrepreneurs' wealth  $Z_t(j)$  after consumption, for  $j = 2, \dots, T-1$ , where we have used Lemma 5. Here  $(K_t(1 - \xi)\tilde{R}_t + H_t((1 - \delta_h)Q_t + r_t))$  is the total return from holding aggregate capital and houses, while  $\frac{K_t(j)}{K_t}$  is the fraction of cohort- $j$ 's wealth in the total. Notice that  $Z_t(T) = 0$  because an age- $T$  entrepreneur consumes all his/her wealth. Equations (B.14), (B.15), and (B.16) define the aggregates. The variable  $N_t$  denotes exogenous worker population.

Equations (B.17)-(B.21) are the firm's first-order conditions with respect to  $k_{c,t}$ ,  $k_{h,t}$ ,  $n_{c,t}$ ,  $n_{h,t}$ , and  $l_t$ , respectively. Equations (B.22)-(B.24) follow from the definitions of  $Y_t$ ,  $Y_{h,t}$ , and  $M_t$ . Equation (B.25) is the no-arbitrage condition. It shows that the initial fundement housing value satisfies

$$Q_0 = \sum_{s=1}^{\infty} \frac{(1 - \delta_h)^{s-1} r_s}{\prod_{j=1}^s \tilde{R}_j}.$$

The predetermined variables for the equilibrium system are  $A_0$ ,  $K_0$ ,  $H_0$ ,  $\{K_0(j)\}_{j=1}^T$ , and  $B_0^g$ .

### B. 2.1 Algorithm for Computing the Dynamics Given $Q_0$

At the beginning of time 0,  $B_0^g$ ,  $A_0$ ,  $H_0$ ,  $K_0$ , and  $\{K_0(j)\}_{j=1}^T$  are known. Given  $Q_0$ , the dynamics are computed as follows.

- (i) Initialize  $t = 0$ . Given  $B_0^g$ ,  $A_0$ ,  $H_0$ ,  $K_0$ ,  $\{K_0(j)\}_{j=1}^T$ , and  $Q_0$ , solve

$$\left( R_0, w_0, p_{L0}, K_{c,0}, K_{h,0}, N_{c,0}, N_{h,0}, Y_0, Y_{h,0}, M_0, Z_0, \{Z_0(j)\}_{j=1}^T \right),$$

by using equations (B.12)-(B.24) in the previous equilibrium system. The total numbers of equations and unknowns are both  $T + 11$ .

- (ii) Given  $B_t^g, A_t, H_t, K_t, \{K_t(j)\}_{j=1}^T, Q_t, R_t, w_t, p_{Lt}, K_{ct}, K_{ht}, N_{ct}, N_{ht}, Y_t, Y_{ht}, M_t, Z_t$ , and  $\{Z_t(j)\}_{j=1}^T$ ,
  - (a) solve  $(B_{t+1}^g, A_{t+1}, H_{t+1}, K_{t+1}, \{K_{t+1}(j)\}_{j=1}^T)$  by using (B.5)-(B.10). The total numbers of equations and unknowns are both  $T + 4$ .
  - (b) given  $(B_{t+1}^g, A_{t+1}, H_{t+1}, K_{t+1}, \{K_{t+1}(j)\}_{j=1}^T)$ , solve  $R_{t+1}, w_{t+1}, p_{Lt+1}, K_{c,t+1}, K_{h,t+1}, N_{c,t+1}, N_{h,t+1}, Y_{t+1}, Y_{h,t+1}, M_{t+1}, Z_{t+1}, \{Z_{t+1}(j)\}_{j=1}^T, Q_{t+1}$  by using time- $(t+1)$  versions of (B.12)-(B.25). The total numbers of equations and unknowns are both  $T + 12$ .

(iii) Set  $t = t + 1$  and go to step (ii).

## B. 2.2 Algorithm for Computing Equilibrium

We use the shooting method. Set a large time horizon  $\bar{T}$  and use the bisection method to compute  $Q_0$  such that the bubbleless equilibrium converges to the balanced growth path.

- (i) Choose two initial values of  $Q_0$ :  $(Q_0^h, Q_0^l)$ .
- (ii) If  $|Q_0^h - Q_0^l| < \epsilon$ , then stop. Otherwise, define  $Q_0 = \frac{Q_0^h + Q_0^l}{2}$ .
- (iii) Given  $B_0^g, A_0, H_0, K_0, \{K_0(j)\}_{j=1}^T$  and  $Q_0$ , solve the system dynamics by using the algorithm in Section B. 2.1.
  - (a) If  $\phi_t \equiv Q_t H_{t+1} / Z_t > 0$  for all  $t = 0, 1, \dots, \bar{T}$ , then set  $Q_0^h = Q_0$  and go to step (ii).
  - (b) If  $\phi_t < 0$  for finite  $t$  (i.e.,  $Q_t$  becomes negative because the initial guess of  $Q_0$  is too low), then set  $Q_0^l = Q_0$  and go to step (ii).
- (iv) Increase  $\bar{T}$  until the solution for  $Q_0$  does not change much. In this case  $\phi_{\bar{T}}$  converges to zero.

## B. 2.3 Bubbleless Balanced Growth

Notice that  $g_r, g_e$ , and  $g_n$  are exogenous growth rates of rent, labor-augmented technology, and population, respectively, and that  $g_l$  is the exogenous declining rate of land quality. We use  $g_x$  to denote the growth rate of a variable  $x_t$ . On a balanced growth path, we have

$$g_A = g_K = g_{K_c} = g_{B^e} = g_Z = g_Y = g_M = (1 + g_e)(1 + g_n) - 1, \quad (\text{B.26})$$

$$g_w = g_e, \quad g_{N_c} = g_n. \quad (\text{B.27})$$

Moreover, the capital return  $R_t, \hat{A}_t$ , and  $\tilde{R}_t$  are constant over time.



It follows from equation (B.25) that the housing price  $Q_t$  grows at the growth rate of rents, i.e.,  $g_Q = g_r$ . By equation (B.18), we have

$$1 = (1 + g_r) (1 - g_l)^{\alpha_l} (1 + g_{K_h})^{\alpha_k - 1} [(1 + g_e) (1 + g_{N_h})]^{1 - \alpha_k - \alpha_l}.$$

By equation (B.20), we have

$$1 + g_w = (1 + g_r) (1 - g_l)^{\alpha_l} (1 + g_{K_h})^{\alpha_k} (1 + g_e)^{1 - \alpha_k - \alpha_l} (1 + g_{N_h})^{-\alpha_k - \alpha_l}.$$

From the above two equations, we can solve for  $g_{K_h}$  and  $g_{N_h}$  :

$$\begin{aligned} g_{N_h} &= \frac{(1 + g_r)^{\frac{1}{\alpha_l}} (1 - g_l)}{1 + g_e} - 1, \\ g_{K_h} &= (1 + g_r)^{\frac{1}{\alpha_l}} (1 - g_l) - 1. \end{aligned}$$

Using the housing production function, we can derive

$$g_{Y_h} = (1 + g_r)^{\frac{1}{\alpha_l} - 1} (1 - g_l) - 1.$$

Thus the growth rate of residential investment  $Q_t Y_{ht}$  is given by

$$(1 + g_r)^{\frac{1}{\alpha_l}} (1 - g_l) - 1.$$

By (B.7),  $g_H = g_{Y_h}$ . It follows from equations (B.5) and (B.21) that

$$g_{p_L} = g_{B^g} = (1 + g_r)^{\frac{1}{\alpha_l}} (1 - g_l) - 1.$$

By the labor market clearing condition,

$$1 = \frac{N_{ct}}{N_t} + \frac{N_{ht}}{N_t}.$$

For a bubbleless balanced growth path to exist, we must have  $g_n \geq g_{N_h}$  as  $g_{N_c} = g_n$ , or

$$1 + g_n \geq \frac{(1 + g_r)^{\frac{1}{\alpha_l}} (1 - g_l)}{1 + g_e}.$$

Under this condition, we deduce that the growth rate of the housing sector (or residential investment  $Q_t Y_{h,t}$ ) is lower than that of the nonhousing sector. Thus  $\phi_t$  converges to zero as  $t \rightarrow \infty$ .

### B. 3 Equilibrium with Stochastic Bubbles

Once the bubble bursts, it never reappears and the equilibrium system is the same as that in Appendix B. 2. Before it bursts, the equilibrium system is also the same as in Appendix B. 2, except for two changes. First, we add superscript + to all endogenous variables in equations (B.5)

through (B.24) to indicate that these variables are in the bubbly state. Second, the no-arbitrage equation (B.25) is replaced by the following equation:

$$\begin{aligned}
& (1 - p_{t+1}) \frac{(R_{t+1}^{h-} \phi_t^+ + \tilde{R}_{t+1}^-(1 - \phi_t^+))}{(R_{t+1}^{h+} \phi_t^+ + \tilde{R}_{t+1}^+(1 - \phi_t^+))} R_{t+1}^{h+} + p_{t+1} R_{t+1}^{h-} \\
&= (1 - p_{t+1}) \frac{(R_{t+1}^{h-} \phi_t^+ + \tilde{R}_{t+1}^-(1 - \phi_t^+))}{(R_{t+1}^{h+} \phi_t^+ + \tilde{R}_{t+1}^+(1 - \phi_t^+))} \tilde{R}_{t+1}^+ + p_{t+1} \tilde{R}_{t+1}^-, \tag{B.28}
\end{aligned}$$

where

$$\begin{aligned}
R_{t+1}^{h+} &\equiv \frac{(1 - \delta_h) Q_{t+1}^+ + r_{t+1}}{Q_t^+}, & R_{t+1}^{h-} &\equiv \frac{(1 - \delta_h) Q_{t+1}^- + r_{t+1}}{Q_t^-}, \\
\tilde{R}_{t+1}^+ &\equiv \frac{R_{t+1}^+ - \xi R^f}{1 - \xi}, & \tilde{R}_{t+1}^- &\equiv \frac{R_{t+1}^- - \xi R^f}{1 - \xi}, & \phi_t^+ &\equiv \frac{Q_t^+ H_{t+1}^+}{Z_t^+}.
\end{aligned}$$

The new no-arbitrage equation (B.28) takes into account the stochastic bubble. The variable  $\phi_t^+$  denotes the portfolio share of the housing investment,  $R_{t+1}^{h+}$  ( $R_{t+1}^{h-}$ ) is the housing return when the bubble persists (bursts), and  $\tilde{R}_{t+1}^+$  ( $\tilde{R}_{t+1}^-$ ) is the effective capital return when the bubble persists (bursts). When the bubble bursts, it will not reappear and  $Q_{t+1}^-$  and  $R_{t+1}^-$  represent the housing price and capital return in the bubbleless equilibrium studied in Appendix B. 2. The solution algorithm is similar to that described in Appendix B. 2 and is omitted here.

We now discuss the bubbly balanced growth path in the long run. According to our calibration, the bubble will never burst in the long run as the bursting probability converges to zero,  $\lim_{t \rightarrow \infty} p_t = 0$ . Thus equation (B.28) reduces to (B.25). On a bubbly balanced growth path, the capital return  $R_t$  and the productivity  $\hat{A}_t$  are constant over time. It follows from (B.25) that the housing growth rate is higher than the rent growth rate and

$$\tilde{R} = \frac{R - \xi R^f}{1 - \xi} = \frac{(1 - \delta_h) Q_{t+1}}{Q_t} = (1 - \delta_h) (1 + g_Q). \tag{B.29}$$

For a bubbly balanced growth path to exist, the growth rate of the housing sector must be the same as that of the nonhousing sector. That is, the growth rate of  $Q_t Y_{ht}$  is equal to  $(1 + g_e)(1 + g_n) - 1$ . Following the same method as in Appendix B. 2, we can derive the growth rates

$$\begin{aligned}
g_A &= g_K = g_{K_c} = g_{K_h} = g_{B^e} = g_Z = g_Y = g_{B^g} \\
&= g_M = g_{p_L} = (1 + g_e)(1 + g_n) - 1, \tag{B.30}
\end{aligned}$$

$$g_w = g_e, \quad g_{N_c} = g_{N_h} = g_n \tag{B.31}$$

$$g_{Y_h} = (1 - g_l)^{\alpha_l} ((1 + g_e)(1 + g_n))^{1 - \alpha_l} - 1, \tag{B.32}$$

$$g_Q = \left( \frac{(1 + g_e)(1 + g_n)}{1 - g_l} \right)^{\alpha_l} - 1. \tag{B.33}$$

Again  $g_H = g_{Y_h}$ . It follows from (B.29) that the bubbly steady-state capital return is

$$R = (1 - \delta_h) (g_Q + 1) (1 - \xi) + \xi R^f.$$

We also have  $\phi_t$  converges to a constant in  $(0, 1)$  as  $t \rightarrow \infty$ .

We need the condition

$$1 + g_Q = \left( \frac{(1 + g_e)(1 + g_n)}{1 - g_l} \right)^{\alpha_l} > 1 + g_r.$$

We also need a condition that the bubbleless capital return is less than the economic growth rate  $(1 + g_e)(1 + g_n)$ , similar to that in the basic model of Section 3. Due to the complexity of the extended model, we will not derive this condition here. We can easily verify it in our numerical solutions.

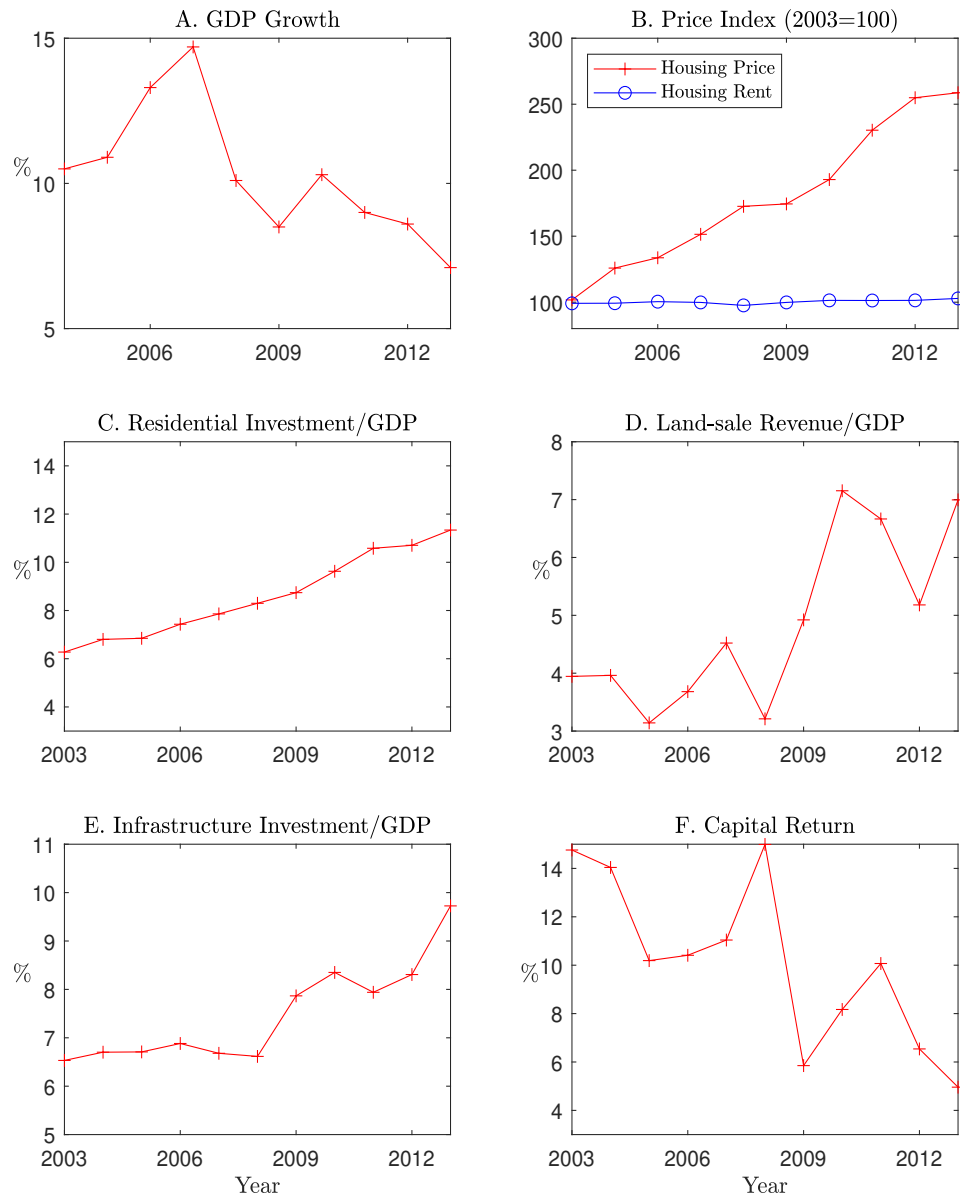


Figure 1: Stylized facts.

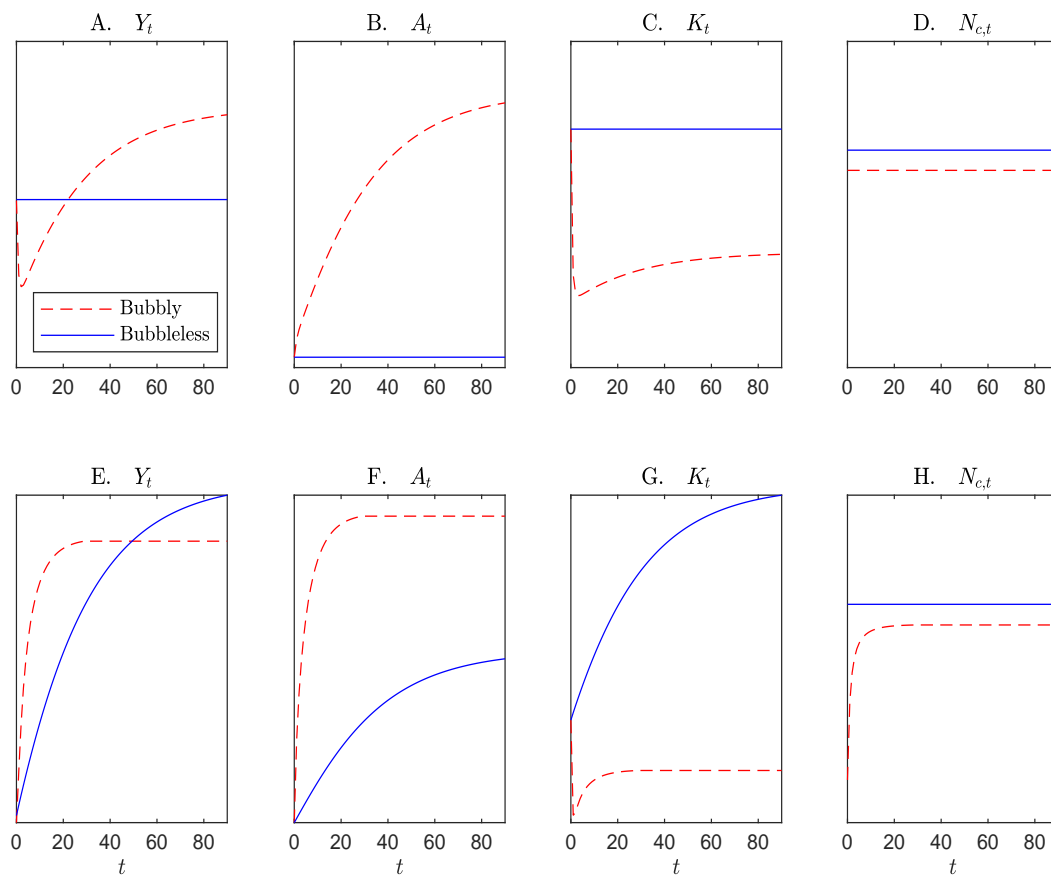


Figure 2: Transitional dynamics.

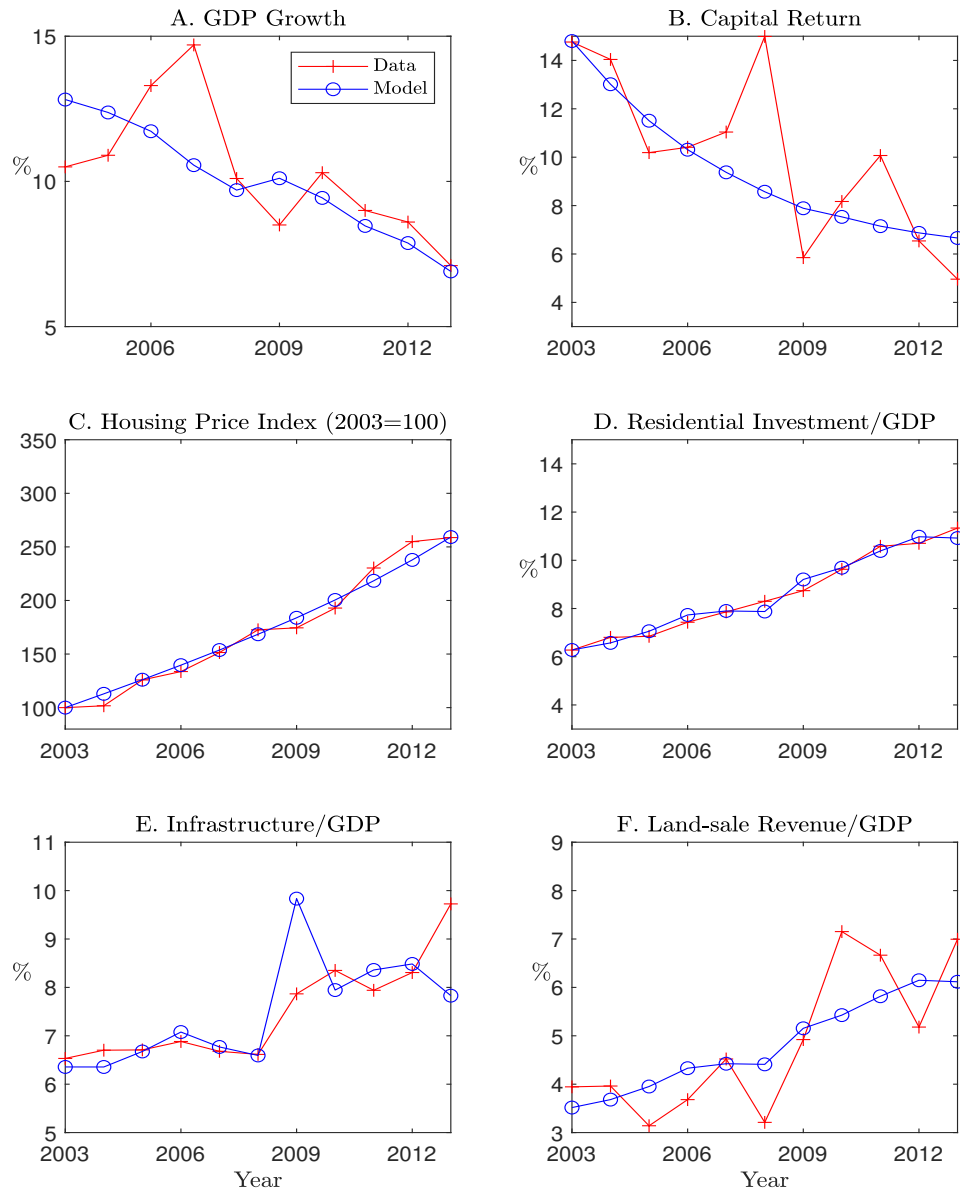


Figure 3: Model results and comparison with the data.

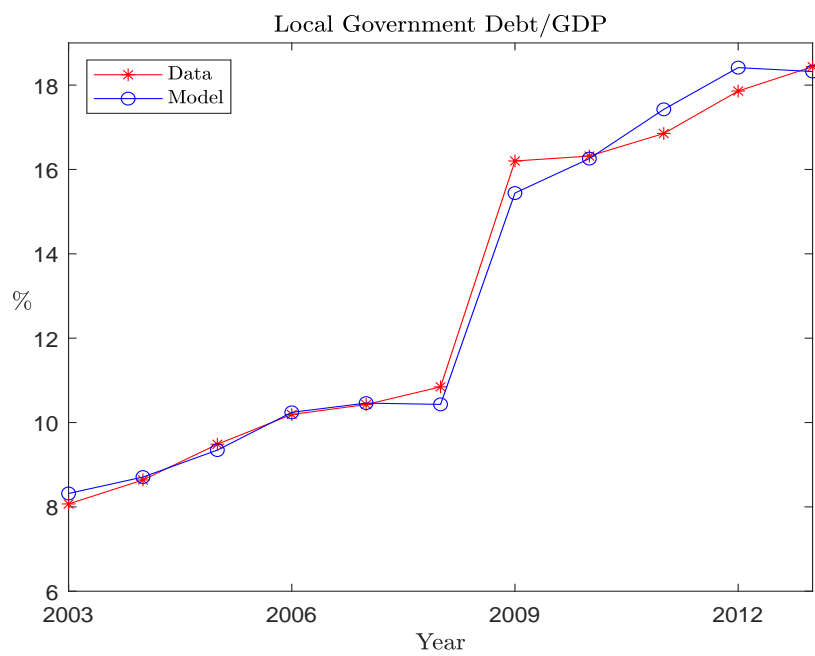


Figure 4: Government debt to GDP Ratio.

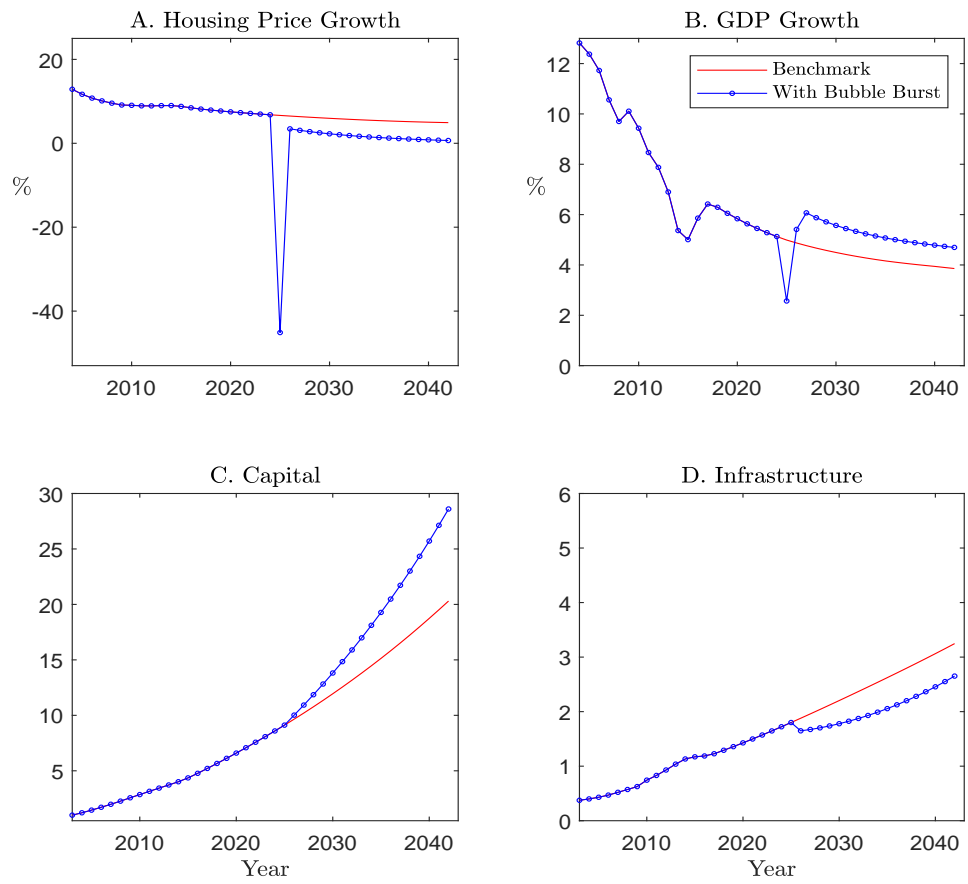


Figure 5: Counterfactual experiment of a future bubble burst.



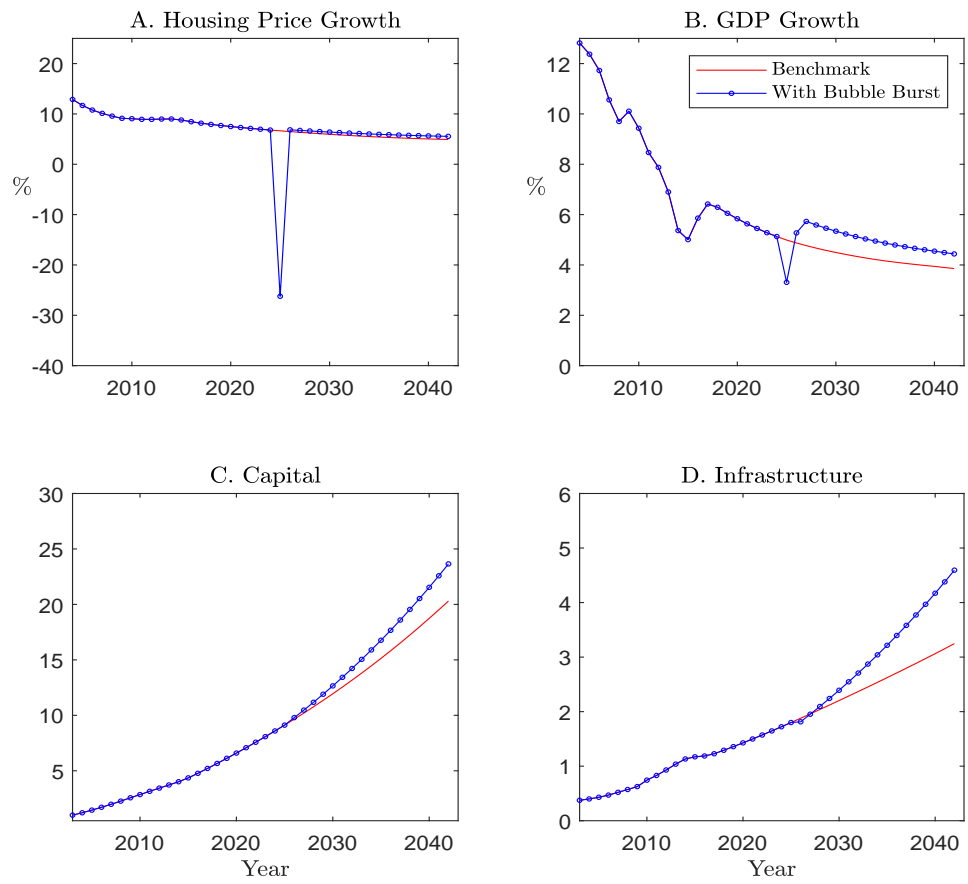


Figure 6: Counterfactual experiment of a future property tax.

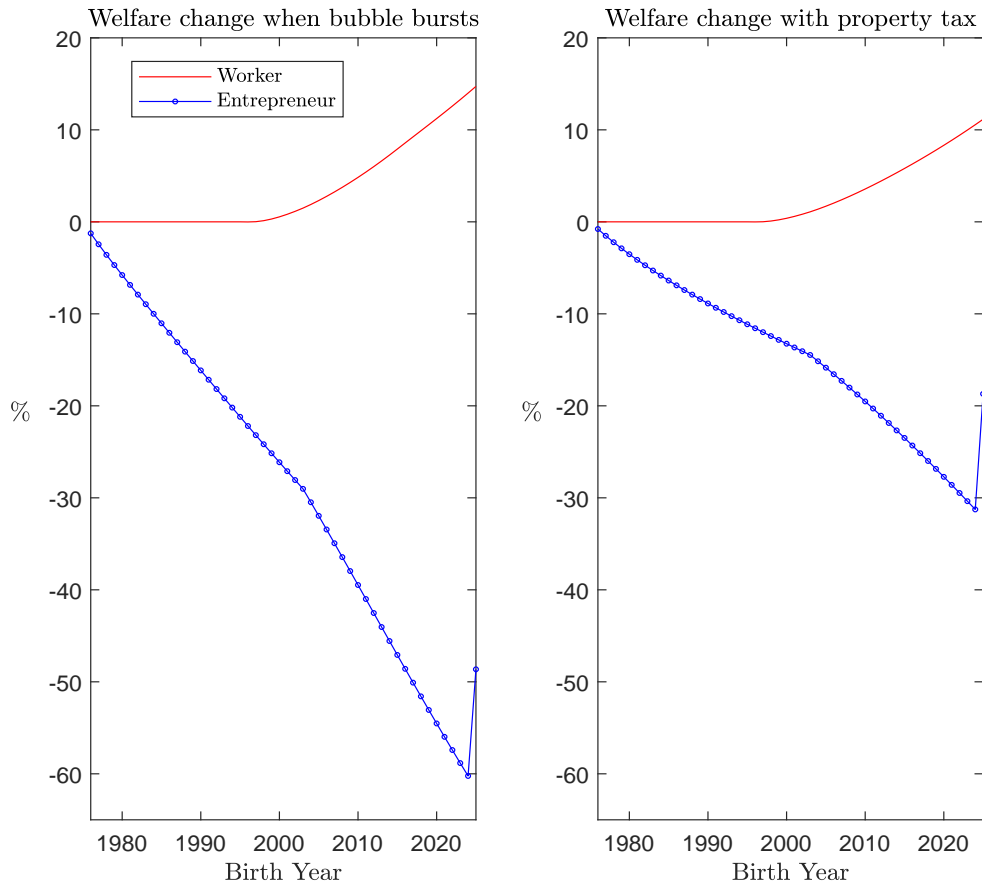


Figure 7: Welfare effects.